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A Fixed-Interest-Rate New Keynesian Model of China*

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Abstract

Nominal interest rates in China has long been controlled by the government, making their changes lagging behind price changes. We model this in a New Keynesian model with a transiently fixed interest rate, and prove that interest rate fixation can magnify model volatility and lead to economic instability. Under the fixed interest rate, the model enters a vicious spiral until monetary policy switches to a flexible interest rate rule, which represents the shadow rate of the economy, determined by discrete (and insufficient) interest rate adjustments and other policy tools. This explains Chinas large business cycle fluctuations over the past decades.

Keyword: Interest rate peg, Chinese economy, New Keynesian Model, Monetary policy, Business cycle

JEL: E31, E32, E42, E43, E52, E58

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1. Introduction

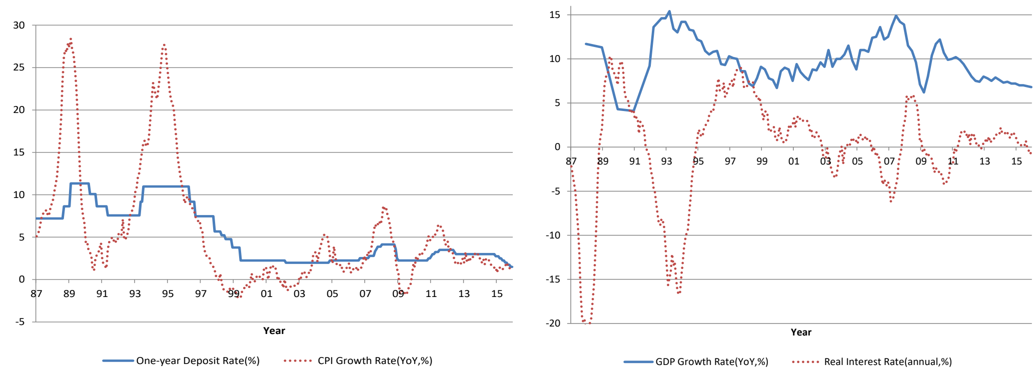
1.1. Interest Rates and the Business Cycle

Since the days of the planned economy, strict administrative controls have been imposed over most interest rates in China, including deposit and lending rates of banks. These rates are not determined in the market, but set by the Chinese government, causing their adjustment to lag behind price change. Figure 1a shows that the inflation rate changes substantially while the nominal interest rate is relatively stable.[1] This leads to sharp fluctuations of the real interest rate and its negative relation with output in Figure 1b. For comparison, in Figure 2, the nominal interest rate in the U.S. is more volatile than inflation before the year 2009, when the former was pushed to the zero level due to the recession; and the relation between output growth and the real interest rate is less pronounced.[2]

A tentative explanation for Figures 1 and 2 is that, given the nominal interest rate unchanged, the real interest rate changes oppositely with inflation and expected inflation. And output is usually positively related with

[1] We represent the nominal interest rate with the benchmark one-year deposit rate instead of the inter-bank market interest rate. As is shown below, when the deposit and lending rates are controlled, the interbank market rate has not much impact on the real economy, as long as bank credit remains the primary financing channel. This condition is satisfied before the recent market-oriented reform of interest rates, especially before the year 2000.

[2] The correlation between YoY output growth and the real interest rate in China is -0.53. The correlation in U.S. data is -0.1 for the subperiod 1987:1–2008:4 and -0.56 for the subperiod 2009:1–2015:4, which reflects that interest rate fixation changes the relation between the two series after 2008. More on this in Section 5.2 and 6.



(a) Nominal Rate and Inflation

(b) Real Rate and Output Growth

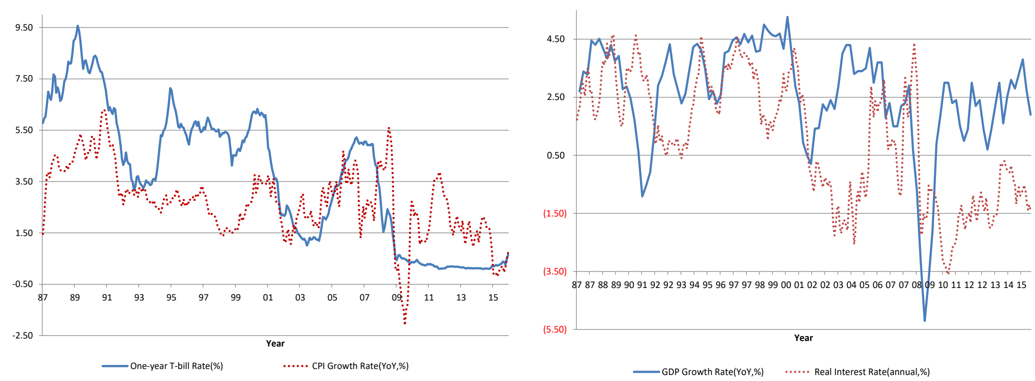
Figure 1: Interest Rates and Business Cycles in China

Notes: To remove seasonal factors in the Chinese data, we use year-over-year growth rates of monthly CPI and quarterly GDP, because the NBS doesn't publish seasonally adjusted data of CPI, and didn't publish those of GDP until 2011. The (realized) real interest rate is constructed as the one-year nominal rate minus the YoY CPI growth rate during the same period.
Sources: CPI (1987:1–2015:12) and GDP (1987–1991; 1992:1–2015:4) come from the National Bureau of Statistics (NBS); one-year deposit rates (1987:1–2015:12) are from the People's Bank of China (PBoC).

inflation, thus inversely related with the real interest rate. The opposite movement between output and the real interest rate brings about a positive feedback mechanism in the economy, which further adds to the volatility of output, inflation, and the real interest rate.

Friedman (1968, p.5) asserts that monetary policy "cannot peg interest rates for more than very limited periods". Woodford (2003, p.46) paraphrases it as follows:

> "With a nominal interest rate that is fixed at a level below the 'natural rate', inflation is generated that increases inflation expectations, which then stimulates demand even further due to the reduction in the real rate, generating even faster inflation, further



(a) Nominal Rate and Inflation

(b) Real Rate and Output Growth

Figure 2: Interest Rates and Business Cycles in U.S.

Sources: One-year treasury bill rates (1987:1-2015:12) are retrieved from Federal Reserve Bank of St. Louis; GDP (1987:1-2015:4) come from U.S. Bureau of Economic Analysis; CPI (1987:1-2015:12) come from U.S. Bureau of Labor Statistics.

> increasing inflation expectations, and so on without bound. The same process should occur with the opposite sign if the interest rate happens to be set above the natural rate; thus any attempt to fix the nominal interest rate would seem almost inevitably to generate severe instability of the inflation rate."

It is implied that the economy may easily fall into a vicious circle under the fixation of nominal interest rates (or inadequate adjustment, i.e., nominal interest rates adjust less than one-for-one to expected inflation), diverging upward or downward until it collapses.

In reality, the Chinese economy indeed experiences drastic fluctuations: the YoY growth rate of monthly CPI ranges from -2.2% to 28.4% in Figure 1a, and that of real GDP from 4.1% to 15.4% in Figure 1b. However, the extreme situation of collapse didn't happen. The reason is that, besides interest rate adjustment, the government also intervenes through other policies. These include official control over bank credit and corporate investment, regulation on land use and real estate prices, enforcement of environmental protection laws, certification of industry access, etc. The overall effect of these measures amounts to a change in the shadow interest rate faced by economic agents. When the shadow rate reaches the level prescribed by a flexible interest rate rule, the stability of the economy can be restored.

1.2. Business Cycles and Monetary Policy in China

In Figure 1a the official deposit rate usually adjusts after a limited period of fixation, when the economy deviates significantly from the steady state. Consider the high inflation periods: the panic buying during 1988–1989, the investment expansion in the early 1990s, and the "overheating" from 2007 to early 2008. In face of high inflation, the government raised nominal interest rates: official deposit and lending rates are raised twice in 1988-89 and 1993 respectively, and six times in 2007. These adjustments relieved the pressure of inflation, but not enough to increase real interest rates. The economy enters an upward spiral. Eventually, the government resorts to administrative means, such as direct control on bank credit and corporate investment. These policies worked effectively to cool down the economy, but also led to excessive macroeconomic volatility, even produced destructive results (the so-called "hard landing").

Similarly, in times of deflation—such as the tightening and readjustment during 1989–1991, the depression from the year 1998 to 2001, and the sharp decline of output growth in the second half of 2008—there are also successive cuts of the nominal interest rate. Benchmark interest rates were lowered three times during 1990-1991, seven times in a row during 1998, and four times in the last three months of 2008. But these delayed adjustments were not sufficient to reduce real interest rates. Real interest rates continued to discourage investment, and thus dampened demand and prolonged the downturn. Meanwhile, the government took quantitative and administrative measures to expand credit and reduce the shadow rate below the official lending rate, to stimulate demand and prevent the economy from a downward spiral. These measures, coupled with external forces, such as export expansion starting from 2002 and the "4 Trillion" infrastructure investment plan from November 2007, brought the economy back on the growth path.

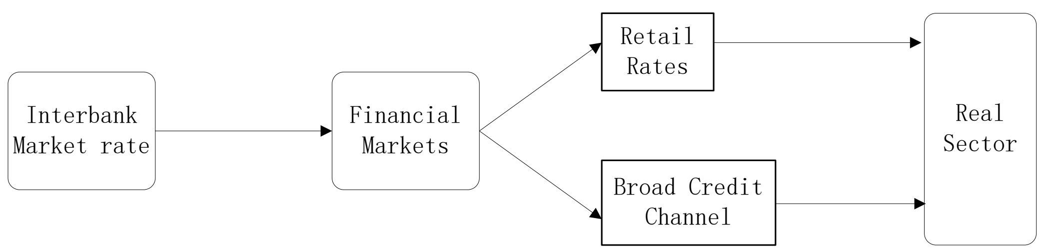


Figure 3: Monetary Policy Transmission in Developed Economy

Interest rate regulations not only lead to economic instability, but also change monetary policy transmission. The typical transmission channel in a textbook (as shown in Figure 3) is that the central bank changes the short-term policy rate in the interbank market, thus altering funding costs of banks; banks accordingly adjust their rates on saving deposits and loans, which then affect the consumption and investment decisions of households and firms. Monetary policy also propagates through credit channels, which changes the balance sheet of the private sector, and therefore affects its borrowing, lending, and spending. The variables most relevant for aggregate spending (including deposit and lending rates, the quantity of loans) are not decided by the central bank, but determined by the market supply and demand. Such a mechanism relies on a well-functioning financial system with adequate instruments and independent institutions.

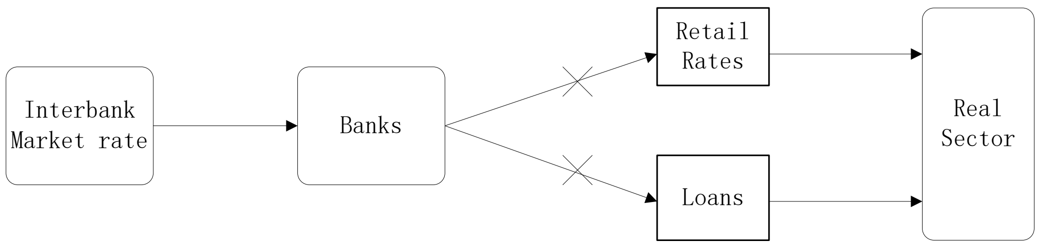


Figure 4: Monetary Policy Transmission in China

Note: Retail interest rates and the quantity of credit in China are not decided by commercial banks, but by the government.

Unlike developed countries, China's monetary and financial system is distorted and underdeveloped. Financial instruments are lacking—to implement policy objectives, the government has to intervene directly by setting loan quotas and retail interest rates of all maturities, an operation too drastic for a smoothly functioning market economy. The interbank money market was established in 1994, and its rates liberalized in 1996; but their impact on the real economy is limited, since commercial banks, the main sources of financing, are tightly controlled. The standard transmission channel of monetary policy is blocked in China (as shown in Figure 4); therefore it is not appropriate to represent monetary policy with a Taylor rule of the interbank market rate, which is the usual practice in DSGE models.

The past few years have seen an acceleration in financial innovation and reform. Direct financing becomes an important channel for companies to raise funds; at the same time emerges a vast and barely regulated shadow banking system, including online and informal financing. The share of bank deposits and loans in nominal assets is decreasing, so that the influence of fixed deposit and lending rates declines.[3] The majority of the system of interest rates have been liberalized, which makes the applicability of a flexible interest rate rule improve. However, complete liberalization of interest rates has not achieved. Official regulations on retail rates were discarded, but only to be replaced by hidden restrictions. An industry self-disciplinary mechanism of market interest rate pricing, established in September 2013, imposes de facto controls on deposit and lending rates. Commercial banks have not gained full autonomy in pricing, and the PBoC continues to intervene in credit allocation. A market-based transmission mechanism from the policy rate to the real sector has yet to be established.

To summarize, China's monetary policy is characterized by the inflexibility of interest rates—a finite period of fixation followed by inadequate adjustment of retail rates; combined with quantitative tools and administrative measures, it prevents the economy from collapsing. To capture this mechanism, we assume a monetary policy rule with a transiently fixed inter-

[3]The change is reflected in a new statistical indicator constructed by the PBoC in 2010: "aggregate financing to the real economy (AFRE)", i.e., the total volume of financing provided by the financial system to the real sector, which is a broad measure of credit and liquidity in the economy. This statistic shows that the proportion of bank loans in AFRE (flow) declines from 92% in 2002 to about 50% in 2013, and then returns to about 90% in 2018 due to the internalizing of "off-balance-sheet" financing. AFRE (stock), an indicator more comparable internationally, shows that the share of bank loans reaches 70% by July 2019, dropping 20 percentage points from that in 2002. See Sheng et al. (2016).

est rate. That is, the nominal interest rate returns to a flexible interest rate rule after a limited period of fixation. We embed this policy in a standard New Keynesian model, and analyze its propagation mechanism, to explain China's business cycle fluctuations. The rest of the paper proceeds as follows: Section 2 reviews the literature; Section 3 establishes the model, Section 4 and 5 carry out the analysis; Section 6 is discussion; Section 7 concludes.

2. Literature Review

Friedman (1968) based his analysis of interest rate pegging in a framework of adaptive expectation. In rational expectation models, the best known result is from Sargent & Wallace (1975), who prove that an exogenously specified interest rate path leads to price indeterminacy. McCallum (1981) and Woodford (2003) argue prices can be determinate for endogenous interest-rate rules (e.g., the Taylor rule), which involve feedback from endogenous state variables to the nominal interest rate.

A special case of the exogenous interest rate is the zero lower bound (ZLB) model, in which the nominal interest rate is fixed at the zero level. The model economy is assumed to be driven to the ZLB by an adverse demand shock, which produces a negative real interest rate. The shock can be a preference shift such as those in Krugman et al. (1998), Woodford & Eggertsson (2003), or the financial deleveraging shock in Eggertsson & Krugman (2012). To ensure unique equilibrium while maintaining rational expectation assumption, the shock is assumed to last for a limited period, after which the economy returns to the steady state.

The fixed-interest-rate model lies in between the exogenous interest rate and the ZLB model. Unlike the ZLB model, in which the policy rate is bounded by zero due to fundamental recessionary shocks, the fixed-interest-rate model augments the interest rate rule with monetary news shocks, to obtain a constant nominal interest rate.[4]. It also differs from the modest policy intervention in Leeper & Zha (2003)—monetary news shocks here are not exogenous, but functions of inflation and output, and can be anticipated by model agents. To exclude equilibrium indeterminacy, the nominal interest rate is fixed for a limited period, and then turns to a flexible interest rate rule satisfying the Taylor principle, that is, the nominal interest rate moves more than one-for-one with the inflation rate. This approach is adopted in Blake (2012), Gali (2009, 2011), and Laseen & Svensson (2011). The difference between our paper and theirs is that they are concerned with the conditional forecasts of DSGE models, while we focus on the volatility of endogenous variables and the propagation mechanism of the model.

Our paper is related to a growing literature that discusses the unusual results obtained in New Keynesian models with fixed or inflexible interest rates. A very partial list includes Eggertsson (2010, 2012), who finds that negative supply shocks are expansionary in the New Keynesian model at the ZLB; Bhattarai et al. (2018), who show that more flexible prices amplify output volatility under passive monetary policy; Christiano et al. (2011), who argue that the fiscal multiplier can be unusually large when the ZLB on the nominal interest rate binds; Del Negro et al. (2015), who document that forward guidance in DSGE models generates implausibly large responses of

[4]The two approaches lead to the same conclusion under sticky prices, but different conclusions under sticky information. See Eggertsson & Garga (2019)

endogenous variables; Carlstrom et al. (2014), who compare fiscal multipliers under a stochastic interest rate peg versus a deterministic one; Carlstrom et al. (2015), who study the "sign reversals" associated with interest rate pegs; and Kiley (2016), who compares sticky prices and sticky information models in a ZLB environment. The difference between our paper and these authors' is that we concentrate on the relation between interest rate fixation and the volatility of model varibles.

Our paper applies the fixed-interest-rate New Keynesian model to the Chinese economy. Although there exist a number of papers investigating China's monetary policy or macroeconomic fluctuation, most of them assume a flexible interest rate rule, or a monetary quantity rule, which implies a flexible interest rate. Only a few of them take seriously the regulation of nominal interest rates. One is He & Wang (2011), who construct a dual-track interest rate model, and study how the change of regulated interest rates affects liberalized interest rates. However, their model does not include goods markets and cannot analyze the feedback between interest rates and endogenous variables such as output and inflation. The other is Chen et al. (2014), who introduce "distortionary taxes" to characterize interest rate regulation, and explore its impact on the structural imbalance of aggregate demand. But their model focuses on the medium and long term; it does not investigate short-term fluctuations, nor the situation when the nominal interest rate is higher than the steady state.

3. The Model

Our analysis is based on the textbook NK model, which can be found in Gali (2015) and Walsh (2017). The linearized version of the model includes the dynamic IS equation and the Phillips curve:

$$\widetilde{y}_t = E_t\widetilde{y}_{t+1} - \frac{1}{\sigma}\left[i_t - E_t\pi_{t+1} - \rho\right] - \frac{1+\varphi}{\sigma+\varphi}\left(a_{t+1} - a_t\right) + \frac{1}{\sigma}\left(1-\rho_z\right)z_t \quad (1)$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa\widetilde{y}_t \quad (2)$$

Here i_t denotes the (log) nominal interest rate, π_t is inflation, $\widetilde{y}_t$ is the output gap, defined as the log deviation of output (y_t) from its flexible price level (natural output, y_t^n), and E_t is an expectation operator. Parameter σ is the inverse elasticity of intertemporal substitution, φ determines the curvature of the disutility of labor, β is the subjective discount factor, and $\rho = -\log\beta$. The symbol κ is a composite parameter (defined in Appendix A) measuring the slope of the Phillips curve. The symbols a_t and z_t represent exogenous shocks: a_t is a technology shock obeying an AR (1) process, with $0 < \rho_a < 1$ and $\varepsilon_t^a \sim N(0, \sigma_a^2)$; z_t affects the discount rate and satisfies $z_t = \rho_z z_{t-1} + \varepsilon_t^z$, where $0 < \rho_z < 1$ and $\varepsilon_t^z \sim N(0, \sigma_z^2)$.

Monetary policy is characterized by the following rule:

$$i_t = \begin{cases} i_t^*, & t > \bar{t}, \\ \bar{d}, & t \leq \bar{t}. \end{cases} \quad (3)$$

When $t \leq \bar{t}$, the nominal interest rate is fixed to the level of $\bar{d}$. After $\bar{t}$, monetary policy switches to a flexible interest rate regime, characterized by $i_t^* = \rho + \phi_\pi\pi_t + \phi_y\widetilde{y}_t + u_t$, where $\phi_\pi > 1$, $0 < \phi_y < 1$, and u_t follows an AR(1) process, with $0 < \rho_u < 1$ and $\varepsilon_t^u \sim N(0, \sigma_u^2)$.

As discussed in section 1, this rule is a characterization of the economic policy in China. After a finite period of interest rate fixation, the economy may deviate significantly from the steady state. The government then reacts with a combination of various policy tools (such as interest rate adjustments, credit control, and other administrative measures), which changes the shadow interest rate in the economy. The changing shadow rate corresponds to i_t^* in the flexible interest rate rule, which guarantees unique equilibrium of the model.

4. Impulse Response Analysis

The parameter values are calibrated as in chapter 3 of Gali (2015). For preference parameters, $\beta = 0.99$, $\varphi = 1$, and $\sigma = 1$. The values of deep parameters imply the composite parameter $\kappa = 0.1717$. The response coefficients of the nominal interest rate to inflation (ϕ_π) and the output gap (ϕ_y) are 1.5 and 0.5/4, respectively. The persistence parameter (ρ_u) of the interest rate shock (u_t) is 0.5, and the standard deviation (σ_u) is 0.0025. The persistence parameter (ρ_z) of the discount rate shock (z_t) is 0.5, and the standard deviation (σ_z) is 0.005.

4.1. The Effects of a Discount Rate Shock

Consider a one standard deviation negative shock to the discount rate. The economy stays at the steady state until period $t = 1$, when it is subject to an exogenous shock. For the fixed rate case, the nominal interest rate is fixed at its steady-state value during periods 1 to $\bar{t}$. From $t = \bar{t}+1$, monetary policy switches to a flexible interest rate regime.

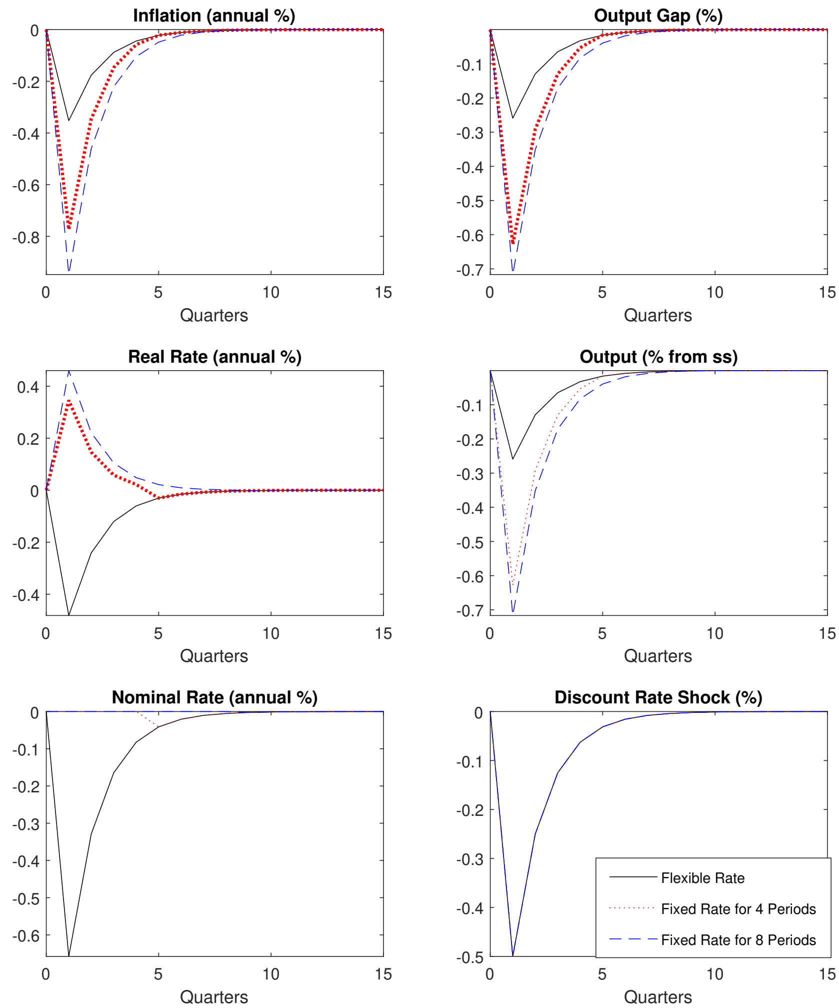


Figure 5: Dynamic Responses to a Discount Rate Shock Under Flexible and Fixed Interest Rates

Notes: This figure plots the impulse responses to a one-unit decrease in the discount rate for different durations of an interest rate peg. The figure shows interest rate fixation increases the volatility of model variables.

Figure 5 compares the impulse responses under different monetary policy regimes, where solid lines denote a flexible interest rate regime, dotted lines correspond to a 4-period fixed rate, and dashed lines to an 8-period fix. All variables are expressed in the log deviation from steady state. Since each period corresponds to one quarter, a one standard deviation decrease of z_t amounts to a 2% drop in the annual discount rate. Inflation, the nominal and real interest rates in Figure 5 (and in all subsequent figures) are expressed in annual rates, that is, quarterly values multiplied by 4.

The responses under a flexible rate regime are identical to that in Gali (2015). The negative discount rate shock represents a decline in the willingness of current consumption, and thus reduces aggregate demand, which decreases inflation and the output gap. The response of output is the same as that of the output gap, because preference shocks do not affect natural output. When the monetary policy rule satisfies the Taylor principle, the nominal interest rate falls more than inflation, which reduces the real interest rate, therefore mitigating the decline of inflation and output.[5]

The fluctuation is more drastic under a fixed interest rate. When inflation falls, the real interest rate increases rather than decreases, which reduces output and the output gap by a greater margin, thus reducing inflation even further, etc. Without a constraint on this self-reinforcing process,

[5]Defining the natural interest rate $r_t^n = \rho + (1 - \rho_z)\, z_t - \sigma\psi_{ya}\,(1 - \rho_a)\, a_t$, and solving equation (1), we obtain $\widetilde{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty} E_t\left\{r_{t+k} - r_{t+k}^n\right\}$. When z_t drops, r_t^n decreases, which reduces $\widetilde{y}_t$. The decline of $\widetilde{y}_t$ is partially offset by the decrease of r_t, which occurs under the Taylor rule. Solving (2) forward, we obtain $\pi_t = \kappa\sum_{k=0}^{\infty}\beta^k E_t\left\{\widetilde{y}_{t+k}\right\}$, which shows that the decline of π_t is also mitigated, due to the reduction of $\widetilde{y}_t$.

the economy will enter a vicious spiral and eventually collapse. This is the "accumulation process" analyzed by Wicksell and Friedman. This spiral can be broken by the expectation of turning to a flexible interest rate regime in the future. After the fixed period, lower inflation means a lower nominal interest rate, which pushes up consumption and output. Rational households and firms take this into account when making decisions, smooth consumption between periods of the fixed and flexible rates, and determine optimal paths of inflation and output.

Comparing the impulse responses of the flexible and fixed interest rate regime, we can conclude that interest rate fixation increases volatility of the economy. The key mechanism is that the real interest rate moves oppositely with inflation, which generates a positive feedback mechanism: the decline of inflation results in a higher real interest rate, which in turn leads to lower inflation. Comparing the impulse responses under a 4-period and an 8-period fixed rate, we can also conclude that the longer the interest rate is fixed, the greater is the volatility of model variables.

4.2. The Effects of a Technology Shock

Now consider a one standard deviation positive shock to the technology level (a_t) at $t = 1$. Let the persistence parameter $\rho_a = 0.5$, and the standard deviation $\sigma_t^a = 0.01$. Figure 6 compares the responses under the regimes of a flexible rate, a 4-period and an 8-period fixed rate. As before, solid lines denote the flexible rate regime, dotted lines correspond to a 4-period fix, and dashed lines to an 8-period fix.

Under the flexible rate regime, like in Gali (2015), the increase of a_t leads to the decline of inflation and the output gap. The response of output is

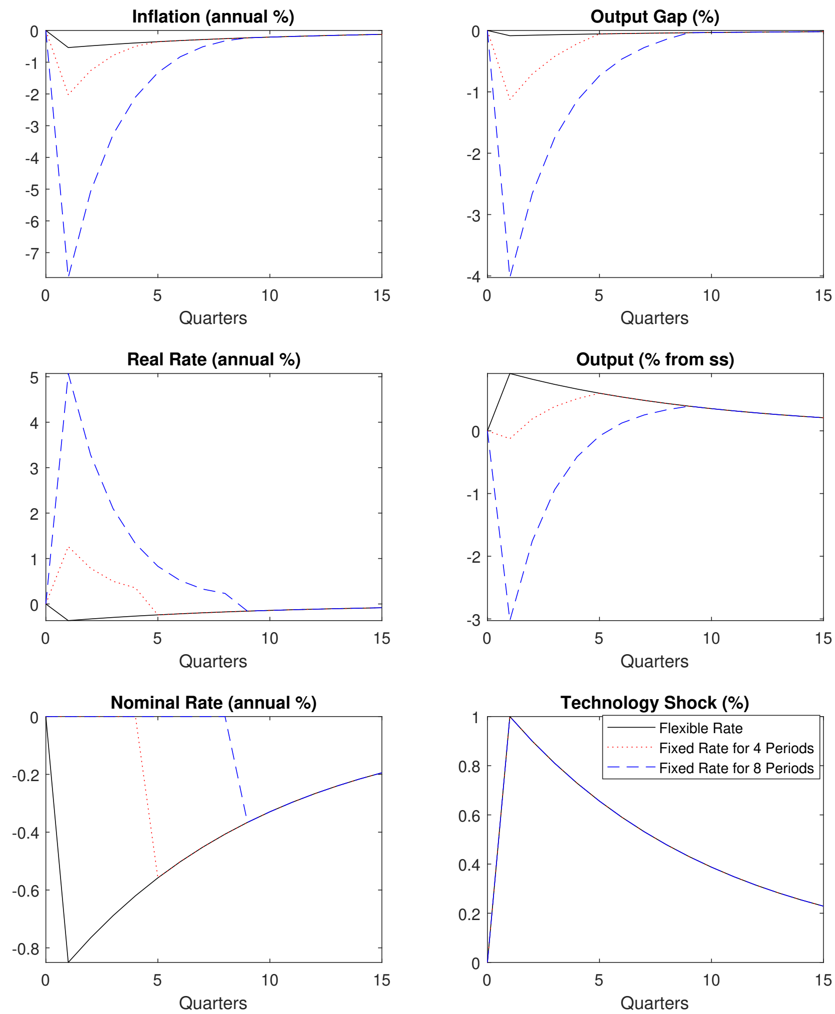


Figure 6: Dynamic Responses to a Technology Shock Under Flexible and Fixed Interest Rates

Notes: This figure plots the impulse responses to a one-unit increase in technology for different durations of an interest rate peg. The figure shows interest rate fixation generally increases the volatility of model variables.

different from that of the output gap, since natural output changes under the technology shock. The decline of the output gap is offset by an increase in natural output caused by the positive technology shock, which thereby raises output level. With the Taylor rule in place, the nominal interest rate declines more than one-for-one with inflation, so that the real interest rate declines, which mitigates the decline of inflation and the output gap, and contributes to the increase of output.

Conversely, under the fixed rate regime, a decrease of inflation leads to an increase in the real interest rate, which further decreases inflation and the output gap. Figure 6 shows that the volatility of both variables increases under a fixed interest rate regime, and the longer is the nominal interest rate fixed, the higher is the volatility. The increase in the real interest rate reduces the output gap by a higher margin, which may exceed the increase of natural output, and lead to the decline of output. That is, the response of output to a technology shock may change its sign under interest rate fixation.[6] Despite this, output declines more as the fixed period extends. Generally speaking, interest rate fixation increases output volatility as well as that of inflation and the output gap.

5. Simulation with All Shocks

In the analysis above, we assume that the nominal interest rate is fixed at its steady-state value and examine model property using impulse response

[6] In the words of Garin et al. (2019), "The basic New Keynesian model predicts that positive supply shocks are less expansionary" under an interest rate peg "compared to periods of active monetary policy".

functions, which are the reactions to a shock hitting the model at the steady state. In reality, nominal interest rates tend to be fixed at levels different from the steady state. To investigate this situation, we run simulations with all shocks hitting the economy.

5.1. Fixed Rates at Different Levels off the Steady State

Figure 7 compares two scenarios when the nominal interest rate is set above the steady-state level by 4% and 2%, respectively. Before period $t = 0$, and after period $\bar{t}$ the economy stochastically fluctuates subject to all three shocks (discount rate, monetary and technology shocks), and monetary policy follows the Taylor rule; from period $t = 1$ to $\bar{t}$, the nominal interest rate is fixed at its value of $t = 0$. Here, $\bar{t}$ equals 5, corresponding to an interest rate fix of 6 periods. The red lines with '+' markers correspond to the simulation under a flexible interest rate, and the green lines with '*' markers correspond to the simulation under a fixed interest rate.

Comparing the simulations under flexible and fixed rates, we can confirm previous results of the impulse response: inflation and output volatility increases under interest rate fixation. During the fixed period, model variables display a tendency to diverge from the steady state, and the economy enters a spiral of depression and deflation. Output and inflation remain below the steady state until monetary policy returns to the flexible interest rate regime. Obviously, output and inflation will enter an upward spiral in the opposite case, when the nominal interest rate is set to a level lower than the steady state. The fixation of the nominal interest rate is equivalent to a series of anticipated monetary shocks, measured by the difference between fixed rates and the rates determined by the flexible policy rule in the first row of Figure

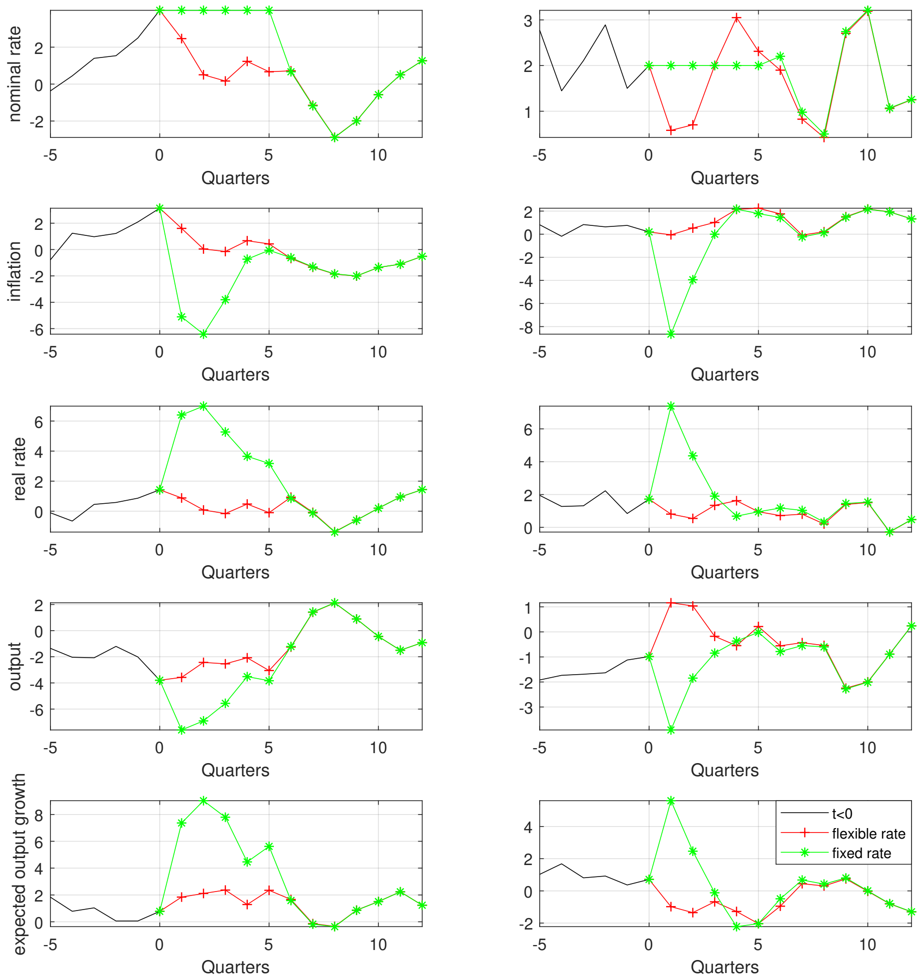


Figure 7: Simulation when the Fixed Rate is Higher than Steady State by 4% (Left) and 2% (Right)

Notes: Inflation and the two interest rates are annualized percentage points, output is the percent deviation from steady state, and expected output growth is the percent change from the previous quarter. The figure shows that model variables in the left column are more volatile, when the fixed rate in the left column deviates farther from the steady state than that in the right column.

7. These shocks override the other shocks (i.e., discount rate and technology shocks), causing model variables to move in one direction until monetary policy returns to the flexible rate regime. We can expect that with the extension of the fixed period, the model economy will deviate farther from the steady state.

Comparing the left and right columns, we can conclude that when the nominal interest rate is fixed at a level farther from steady state, model variables will become more volatile. The volatility of the model economy in Figure 7 can be measured by how much output and inflation drop when the nominal rate is fixed above the steady state. During the fixed period, output in the left column drops 5.5% on average from the steady state, more than 1.4% in the right column, and inflation drops 3.2% on average in the left column, more than 1.7% in the right column. Note the maximum drop of inflation in the left column is 6.4%, less than 8.7% in the right column, because the initial inflation at the beginning of the fixed period is higher in the left column, due to a higher nominal interest rate combined with a relatively stable real interest rate. A more reasonable measure of the tightening effect of a fixed rate above the steady state is how much model variables drop from their initial values at the beginning of the fixed period. For this statistic, output in the left column drops 1.6% on average, more than 0.4% in the right column, and inflation in the left column drops 6.4% on average, more than 1.9% in the right column. And the maximum drop of inflation from its initial value in the left column is 9.6%, more than 8.8% in the right column.

Figure 7 displays the results from only one simulation, which may be subject to occasional factors. For even when the nominal interest rate is fixed at the same value, the realized values of other endogenous variables, which are functions of exogenous shocks, may be different. To cope with such randomness and evaluate the volatility of the two scenarios more strictly, we consider a number of fixed interest rates that fall into the intervals of $[3.99\%, 4.01\%]$ and $[1.99\%, 2.01\%]$. For each interval, select 10 interest rates; for each fixed rate, run 10 simulations. And for each simulation, calculate deviations of model variables from their steady-state and initial values. Then compute the mean of the 10*10 deviations (see Appendix C for details of calculation). Table 1 summarizes the results: the values of the left column are always lower than the corresponding values of the right column. This confirms the results in Figure 7.

Table 1: Deviation of Output and Inflation Under the Fixed Rate

Variables	Percent(%)	Left Column	Right Column
Output	Mean deviation from SS	-4.7	-2.0
	Mean deviation from its initial value	-1.4	-0.6
	Maximum deviation from SS	-1.2	-0.5
	Maximum deviation from its initial value	-3.7	-1.8
Inflation	Mean deviation from SS	-6.8	-3.5
	Mean deviation from its initial value	-4.8	-3.2
	Maximum deviation from SS	-3.4	-2.0
	Maximum deviation from its initial value	-7.1	-4.3

5.2. Fixed Rates Above v.s. Below the Steady State

Figure 8 tries to replicate Figure 1. We simulate two scenarios when the fixed interest rate is higher and lower than the steady-state level by 4%,

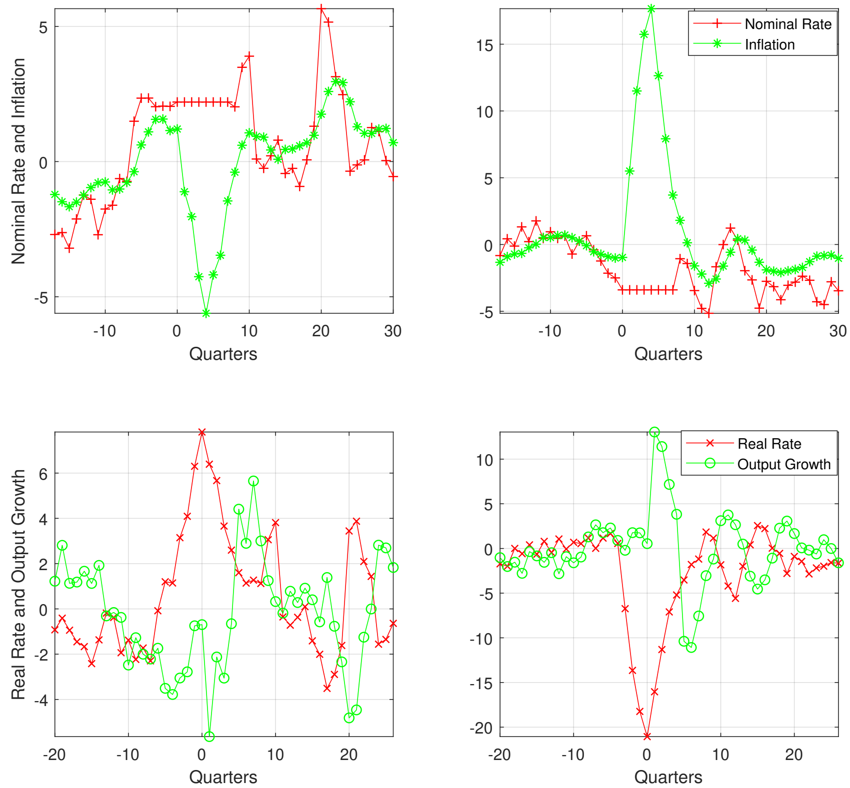


Figure 8: Simulation when the Fixed Rate is Above (Left) and Below (Right) Steady State

Notes: The left column considers a fixed interest rate higher than its steady-state level, while the right column considers a fixed rate lower than the steady state. To be consistent with Figure 1, the nominal and real interest rates are annualized percentage points, inflation and output growth are year-over-year growth rates of price and output level in percentage points. The figure shows that model variables in the two columns move in opposite directions due to the different fixed values of the nominal interest rate.

respectively. Inflation and output growth are represented by YoY growth rates of price and output level, to be consistent with those in Figure 1. The nominal interest rate is fixed for 8 periods, which roughly equals the average fixed length during the past years in China. The left and right columns show completely different paths of inflation and output. That is, the economy can move in opposite directions, depending on whether the nominal interest rate is fixed above or below the steady state.

The upper row in Figure 8 corresponds to Figure 1a, in which drastic inflation fluctuations accompany each period of interest rate fixation; and the inflation hikes here resemble those in Figure 1a. In Figure 1a, the fixed periods are followed by discrete adjustments of nominal interest rates, which alleviates the magnifying effect of interest rate fixation, and helps the public to form the expectation that monetary policy will turn to the flexible rate regime when economic imbalances are severe. Here in Figure 8, these interest rate adjustments are represented by the flexible interest rate rule, which guarantees that the model reverts toward the steady state after the fixed period. The lower row in Figure 8 mimics Figure 1b, in which real interest rates change negatively with YoY growth rates of output during the fixed period; the negative relation is less clear before and after the fixed period, suggesting that interest rate fixation changes the relation between the two series.

The correlation between YoY output growth and the real interest rate is -0.51 during the fixed period and -0.28 under the flexible rate, roughly comparable to those reported in footnote 2 (see Appendix D for details of calculation). We take this correlation as an indicator reflecting the flexibility of the nominal interest rate. The intuition behind it is that inflation usually lags behind output growth, so high output growth leads to high expected inflation, which means a low real interest rate when combined with an inflexible nominal interest rate, and vice versa.

The negative relation between output growth and the real interest rate seems to contradict the Euler equation $(c_{t+1} - c_t + [\rho + (1 - \rho_z) z_t] = i_t -$

$E_t\pi_{t+1}$, see Appendix A), which predicts expected output growth to be positively related with the real interest rate. This condition holds even under a fixed interest rate, as shown in the third and last rows of Figure 7. The difference is that in Figure 7 output growth rates are expected quarter-over-quarter rates, while in Figure 1b and Figure 8 they are year-over-year rates realized at time t.

6. Discussion

Figure 1a shows that China's inflation displays significant fluctuations, especially before the year 2000. The analysis above, particularly the simulation in Figure 8, proves that introducing interest rate fixation into a standard New Keynesian model can magnify model volatility and explain this phenomenon to a large extent. This result is different from Del Negro et al. (2015), who find that inflation in the United States stays at a low level since the recession in 2007, even the nominal interest rate was fixed at the zero level for an extended period. They name this fact as the "forward guidance puzzle", since it contradicts the New Keynesian model, which predicts powerful effects when the promise to peg future interest rates is credible. This difference can be explained by the change of the shadow interest rate.

Wu & Zhang (2019) introduce the shadow rate into the New Keynesian model to represent the effects of unconventional monetary policies. Although short term policy rates are constrained at the ZLB, the Fed can use other tools to stimulate economic activity. And this changes the shadow interest rate faced by the private sector and stabilizes the economy. Similarly, other than interest rate adjustments, the Chinese government also regulates through quantitative tools, which change the shadow interest rate and stabilize the economy. Without these quantitative tools, the shadow rate will not deviate from the fixed official rate, and the economy cannot be stabilized—this explains the significant fluctuations before 2000.

Figure 1 also shows that China's macroeconomic volatility declines after 2000, which can be seen as the "great moderation" of China.[7] The main reasons include more flexible macroeconomic policies and market-oriented reform of the financial system. On one hand, the authority implements various policy instruments more effectively, including quantitative tools and administrative measures. On the other hand, with the development of shadow banking and direct financing (stock market and bond market), the share of banking in the financial sector decreases, and the interest rates of most financial instruments (excluding saving deposits and loans) have been liberalized. These changes imply that the impact of the control on retail rates declines, and the average flexibility of nominal interest rates improves. The two factors—a more flexible policy and a more liberalized financial system—correspond to the shortening of the fixed period in our model, which measures the flexibility of monetary policy.

[7] The standard deviation of YoY output growth in China declines from 2.80 (1987–2000) to 2.16 (2001–2015). Correspondingly, the correlation between YoY output growth and the real interest rate changes from -0.71 (1987–2000) to -0.29 (2001–2015), reflecting the transition toward a more flexible nominal interest rate.

For comparison, the standard deviation of YoY output growth in U.S. data increases from 1.55 (1987–2008) to 1.94 (2009–2015). Correspondingly, the correlation indicator changes from -0.1 (1987–2008) to -0.56 (2009-2015), reflecting the switch from a flexible to a fixed interest rate regime.

Figures 5 and 6 show that a shorter interest rate peg leads to a more stable economy, which explains the decrease of macroeconomic volatility in China after 2000. This explanation is related to Clarida et al. (2000), who argue that inflation in the U.S. after 1980 was stabilized by a shift from an "indeterminate" to a "determinate" regime. Their explanation is questioned by Cochrane (2007): "$\cdots$ a theory that has absolutely nothing to say about inflation in the 1970s, other than 'inflation is indeterminate, so any value can happen' is surely a bit lacking." The approach in this paper—assuming that monetary policy switches from a fixed to a flexible interest rate regime after a finite period—avoids this criticism.[8]

7. Conclusion

China's monetary policy is characterized by the control of retail deposit and lending rates—a finite period of fixation followed by discrete adjustments by the government. We model this in a textbook New Keynesian model, to explain the large fluctuations of China's macroeconomy.

The main conclusions can be summarized as follows: (1) Under interest rate fixation, contrary to a flexible rate regime, the real interest rate and inflation move in opposite directions, which adds positive feedback in model propagation. (2) Inflation and the output gap become more volatile under interest rate fixation, and the economy becomes more sensitive to external

[8]Cochrane (2017) proposes using fiscal policy to obtain a unique equilibrium in the fixed-interest-rate New Keynesian model. During the past decades, China has undergone fundamental changes in its fiscal system. Whether this change has impact on economic stability, and how it interacts with interest rate liberalization deserve further study.

shocks. (3) The longer is the nominal interest rate fixed, the greater the economic volatility. (4) The farther the fixed rate deviates from the steady state, the more unstable the economy. (5) The nominal interest rate fixed above or below the steady state can lead to completely different paths of inflation and output, driving the economy to a vicious circle of inflation or deflation in opposite directions. (6) We use the correlation between YoY output growth and the real interest rate to measure the flexibility of the nominal interest rate, and find this indicator changes from the subperiod 1987–2000 to 2001–2015 in China, as well as in the U.S. from the subperiod 1987–2008 to 2009–2015. And this explains the change of macroeconomic volatility.

Our paper is the first step toward understanding the unusual fluctuation of China's macroeconomy in a macroeconomic model with rational expectation. Our main conclusions are qualitative. The next step may need more quantitative research with econometric tools, such as estimation and statistical tests. It is also necessary to explore how interest rate fixation interacts with other factors within a full-fledged model with investment and financial sector, like that in Smets and Wouters (2003) and Christiano, Motto and Rostagno (2003). And more work is needed to calculate the shadow rate to summarize the policy stance and reflect the composite effects of monetary policy and administrative means.

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Appendix A. The Model

Appendix A.1. Households

The representative household seeks to maximize the objective function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[e^{z_t} \left(\frac{{C_t}^{1-\sigma}}{1-\sigma} - \frac{{N_t}^{1+\varphi}}{1+\varphi} \right) \right] \tag{A.1}$$

where E_0 is the expectational operator, conditional on information at time 0, $\beta \in (0,1)$ is the discount factor, C_t and N_t denote consumption and labor supply, φ determines the curvature of the disutility of labor, σ is the inverse elasticity of intertemporal substitution, z_t is an exogenous preference shifter, satisfying $z_t = \rho_z z_{t-1} + \varepsilon_t^z$, where $0 < \rho_z < 1$, and $\varepsilon_t^z \sim N(0, \sigma_\tau^2)$.

The period budget constraint takes the form:

$$P_tC_t + B_t \leq B_{t-1}R_{t-1} + W_tN_t + T_t \tag{A.2}$$

for $t = 0, 1, 2, \cdots$, P_t is the price of the consumption good, W_t denotes the nominal wage, B_t represents the quantity of one-period nominally riskless discount bonds purchased in period t and maturing in period $t+1$, R_t is the nominal interest rate paid on bonds held at the end of period t, T_t represents net (nominal) tax collections by the government.

In addition to (A.2), the household is subject to a solvency constraint that prevents it from engaging in Ponzi-type schemes:

$$E_0 \lim_{t\to\infty} \frac{B_{t+1}}{(1+R_0)(1+R_1)\cdots(1+R_t)} \geq 0$$

The optimality conditions implied by the maximization of (A.1) subject to (A.2) are given by

$$\beta E_t \left\{ e^{z_{t+1}-z_t} \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{R_t}{\pi_{t+1}} \right\} = 1 \tag{A.3}$$

$$\pi_{t+1} = \frac{P_{t+1}}{P_t} \tag{A.4}$$

$$N_t^{\varphi} C_t^{\sigma} = \frac{W_t}{P_t} \tag{A.5}$$

Appendix A.2. Firms

The final good Y_t is produced by combining a continuum of intermediate goods, indexed by $i \in (0, 1)$, using the technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\varepsilon > 1$, and $Y_t(i)$ denotes the time t input of intermediate good i. The final good producer maximizes its profit:

$$P_t\left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_t(i)Y_t(i)di$$

where $P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}}$. The solution to this problem yields demand curves faced by intermediate-goods producers:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\varepsilon} Y_t$$

Intermediate good $j \in (0,1)$ is produced by a monopolist who uses the following technology:

$$Y_t(i) = e^{a_t} N_t(i) \tag{A.6}$$

Here a_t denotes a technology shock, and satisfies $a_t = \rho_a a_{t-1} + \varepsilon_t^a$, with $0 < \rho_\tau < 1$, and $\varepsilon_t^a \sim N(0, \sigma_a^2)$.

The firm's real marginal cost is

$$s_t = (1-v)\frac{W_t}{e^{a_t}P_t} \tag{A.7}$$

where $v = 1/\varepsilon$ represents employment subsidy, to correct for distortion caused by monopoly power in steady state.

Assume firms set prices according to a variant of the mechanism spelled out in Calvo (1983). In each period, an intermediate goods firm faces a constant probability, $1-\theta$, of being able to re-optimize its nominal price $P_t(i)$. The ability to re-optimize prices is independent across firms and time. Thus, in each period a measure $1-\theta$ of producers reset their prices, while a fraction θ keep their prices unchanged.

Appendix A.2.1. Optimal Price Setting

A firm reoptimizing in period t will choose the price $\widetilde{P}_t$ that maximizes the current market value of the profits generated while that price remains effective. This corresponds to solving the following problem:

$$\max_{\widetilde{P}} E_t \sum_{j=0}^{\infty} \mu_{t+j}\beta^j\theta^j \left[\widetilde{P}_t Y_{t+j}(i) - P_{t+j}s_{t+j}Y_{t+j}(i)\right]$$
$$= \max_{\widetilde{P}} E_t \sum_{j=0}^{\infty} (\beta\theta)^j \mu_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[\widetilde{P}_t^{1-\varepsilon} - P_{t+j}s_{t+j}\widetilde{P}_t^{-\varepsilon}\right],$$

where μ_{t+j} denotes the state-contingent marginal value of one unit of money to a household. The second equation results from substituting out $Y_{t+j}(i)$ using the demand equation $Y_{t+j}(i) = \left[\frac{\widetilde{P}_t}{P_{t+j}}\right]^{-\varepsilon} Y_{t+j}$.

The optimality condition associated with the problem above takes the form:

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j \mu_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[(1-\varepsilon)\widetilde{P}_t^{-\varepsilon} + \varepsilon P_{t+j}s_{t+j}\widetilde{P}_t^{-\varepsilon-1}\right]$$
$$= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{u'(C_{t+j})}{P_{t+j}} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\widetilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon-1}s_{t+j}\right]$$
$$= E_t \sum_{j=0}^{\infty} (\beta\theta)^j \frac{u'(C_{t+j})}{P_{t+j}} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\widetilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon-1}s_{t+j}\right]$$
$$= E_t \sum_{j=0}^{\infty} (\beta\theta)^j Y_{t+j}^{1-\sigma} P_{t+j}^{\varepsilon} \left[\widetilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon-1}s_{t+j}\right]$$
$$= E_t \sum_{j=0}^{\infty} (\beta\theta)^j Y_{t+j}^{1-\sigma} (X_{t,j})^{-\varepsilon} \left[\widetilde{p}_t X_{t,j} - \frac{\varepsilon}{\varepsilon-1}s_{t+j}\right] = 0$$

where $\widetilde{p}_t = \frac{\widetilde{P}_t}{P_t}$ denotes the real value of optimal price, $\mu_{t+j} = \frac{u'(C_{t+j})}{P_{t+j}}$, $s_t = (1-v)\frac{W_t}{e^{a_t}P_t} = (1-v)\frac{N_t^{\varphi}C_t^{\sigma}}{e^{a_t}}$, $X_{t,j} = \begin{cases} \dfrac{1}{\pi_{t+j}\pi_{t+j-1}\dots\pi_{t+1}} & j \geq 1 \\ 1 & j = 0 \end{cases}$.

Solve for $\widetilde{p}_t$, we obtain

$$\widetilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j Y_{t+j}^{1-\sigma} (X_{t,j})^{-\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j Y_{t+j}^{1-\sigma} (X_{t,j})^{1-\varepsilon}} = \frac{K_t}{F_t} \tag{A.8}$$

Here,

$$\begin{aligned} K_t &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j Y_{t+j}^{1-\sigma} (X_{t,j})^{-\varepsilon} \frac{\varepsilon}{\varepsilon - 1} s_{t+j} \\ &= \frac{\varepsilon}{\varepsilon - 1}(1 - v)\frac{N_t^{\varphi} Y_t}{e^{a_t}} + \beta\theta E_t {\pi_{t+1}}^{\varepsilon} K_{t+1} \\ &= \frac{N_t^{\varphi} Y_t}{e^{a_t}} + \beta\theta E_t {\pi_{t+1}}^{\varepsilon} K_{t+1} \end{aligned} \tag{A.9}$$

and

$$\begin{aligned} F_t &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j Y_{t+j}^{1-\sigma} (X_{t,j})^{1-\varepsilon} \\ &= Y_t^{1-\sigma} + \beta\theta E_t {\pi_{t+1}}^{\varepsilon - 1} F_{t+1} \end{aligned} \tag{A.10}$$

Appendix A.2.2. Aggregate Price Dynamics and Price Dispersion

The aggregate price level:

$$\begin{aligned} P_t &= \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \\ &= \left[(1-\theta)\widetilde{P}_t^{1-\varepsilon} + \int P_{i,t}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \\ &= \left[(1-\theta)\widetilde{P}_t^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \end{aligned}$$

where the second equality follows the fact that all firms resetting prices will choose an identical price $\widetilde{P}$, and the third equality follows from the fact that the distribution of prices among firms not adjusting in period t corresponds to the distribution of effective prices in period $t-1$, though with total mass reduced to θ. Dividing both sides by P_t yields $\widetilde{p}_t = \left[\frac{1-\theta{\pi_t}^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}}$. Combining with (A.8), we obtain

$$\frac{K_t}{F_t} = \left[\frac{1-\theta{\pi_t}^{(\varepsilon-1)}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \tag{A.11}$$

We follow Yun(1996) to derive the relation between output (Y_t), aggregate inputs (N_t, A_t) and the distribution of resources. Defining $Y_t^* = \int_0^1 Y_{i,t}di = \int_0^1 A_t N_{i,t}di = A_t N_t$, and substituting with $Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\varepsilon} Y_t$ yields

$$Y_t^* = \int_0^1 Y_{i,t}di = Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} di = Y_t P_t^{\varepsilon} \int_0^1 (P_{i,t})^{-\varepsilon} di = Y_t P_t^{\varepsilon} (P_t^*)^{-\varepsilon}$$

Thus, we obtain the relation:

$$Y_t = \left(\frac{P_t^*}{P}\right)^{\varepsilon} Y_t^* = p_t^* A_t N_t \tag{A.12}$$

Here,

$$p_t^* = \left[(1-\theta)\left(\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\pi_t^{\varepsilon}}{p_{t-1}^*}\right]^{-1} \tag{A.13}$$

where p_t^* measures efficiency distortion, and satisfies the condition:

$$p_t^* \begin{cases} = 1, \ P_{i,t} = P_{j,t} & \text{for all } i,j \\ \leq 1 & \text{otherwise} \end{cases}$$

Appendix A.3. Market Clearing and Monetary Policy

The aggregate resource constraint is

$$Y_t = C_t \tag{A.14}$$

Assume that monetary policy follows

$$R_t = \begin{cases} Z_t & t > \bar{t} \\ \bar{d} & t = 1, \cdots, \bar{t} \end{cases} \tag{A.15}$$

where $Z_t = R\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_y} e^{\varepsilon_t^v}$ is the flexible interest rate determined by the Taylor rule, Y and π are output and inflation of the steady state, $\phi_\pi > 1$, $0 < \phi_\pi < 1$, and $\varepsilon_t^v \sim N(0, \sigma_v^2)$. The nominal interest rate is set to the level $\bar{d}$ at periods $t = 1, \cdots, k$. When $t > k$, monetary policy switches back to the Taylor rule.

Appendix A.4. Natural Output and Log Linearization

Assuming that inflation $\pi = 1$ at the steady state, we obtain $R = \frac{1}{\beta}$, and $F = K = \frac{1}{1-\beta\theta}$. At the zero inflation steady state, relative price distortion is eliminated, i.e., $p^* = 1$.

Log-linearizing (A.13), we obtain the equilibrium law of motion for p_t^* near the steady state, $\widehat{p}_t^* \approx \theta\widehat{p}_{t-1}^* + 0 \times \pi_t = \theta\widehat{p}_{t-1}^*$. Assume $\widehat{p}_0^* = 0$, and this means $\widehat{p}_t^* \approx 0$, which further means $p_t^* \approx 1$. Substitute it into (A.12), we obtain that $Y_t \approx A_t N_t$ holds near the steady state. Taking logs, we obtain

$$y_t \approx a_t + n_t \tag{A.16}$$

Defining the natural level of output, denoted by Y_t^n, as the equilibrium level of output under flexible prices ($p_t^* = 1$). From (A.12),

$$y_t^n = a_t + n_t \tag{A.17}$$

From the equilibrium condition of the labor market and (A.5), we obtain $N_t^{\varphi} {C_t}^{\sigma} = A_t$ (only holds when there is no relative price distortion), or $\sigma c_t +$

$\varphi n_t = a_t$ (in logged form). Combining it with (A.17), we obtain

$$y_t^n = \frac{1+\varphi}{\sigma+\varphi} a_t \tag{A.18}$$

Log-linearizing (A.9)–(A.10) yields

$$\widehat{K}_t = \beta\theta E_t \left(\varepsilon\widehat{\pi}_{t+1} + \widehat{K}_{t+1}\right) + (1-\beta\theta)\left[\widehat{Y}_t + \varphi\widehat{n}_t - \widehat{a}_t\right]$$

$$\widehat{F}_t = \beta\theta E_t \left((\varepsilon-1)\widehat{\pi}_{t+1} + \widehat{F}_{t+1}\right) + (1-\beta\theta)(1-\sigma)\widehat{Y}_t$$

$$\widehat{K}_t - \widehat{F}_t = \frac{\theta}{1-\theta}\widehat{\pi}_t$$

Eliminating $\widehat{K}_t$ and $\widehat{F}_t$, we obtain

$$\widehat{\pi}_t = \beta\widehat{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)\left[(\sigma+\varphi)\widehat{n}_t + (\sigma-1)a_t\right]}{\theta} \tag{A.19}$$

Substituting (A.16) and (A.18) into (A.19), we obtain equation (2) in the main text

$$\widehat{\pi}_t = \beta\widehat{\pi}_{t+1} + \kappa\widetilde{y}_t \tag{A.20}$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)(\sigma+\varphi)}{\theta}$, $\widetilde{y}_t = y_t - y_t^n = a_t + n_t - \left(a_t + \frac{(1-\sigma)a_t}{\sigma+\varphi}\right) = \widehat{n}_t - \frac{(1-\sigma)\widehat{a}_t}{\sigma+\varphi}$.

Log-linearizing (A.3) around the steady state yields

$$\widehat{c}_t = E_t\widehat{c}_{t+1} - \frac{1}{\sigma}\left(\widehat{i}_t - E_t\widehat{\pi}_{t+1}\right) + \frac{1}{\sigma}(z_t - z_{t+1})$$

Rewriting it in terms of the output gap, we obtain equation (1) in the main text:

$$\widetilde{y}_t = E_t\widetilde{y}_{t+1} - \frac{1}{\sigma}\left[i_t - E_t\pi_{t+1} - \rho\right] - \frac{1+\varphi}{\sigma+\varphi}(a_{t+1} - a_t) + \frac{1}{\sigma}(1-\rho_z) z_t \tag{A.21}$$

Log-linearizing (A.15) yields

$$\widehat{i}_t = \begin{cases} \phi_\pi\widehat{\pi}_t + \phi_y\widehat{y}_t + \varepsilon_t^v & t > \bar{t} \\ 0 & t = 1, \cdots, \bar{t} \end{cases} \tag{A.22}$$

which corresponds to equation (3) in the main text.

Appendix B. Calibration

Parameter values are consistent with those in chapter 3 of Gali (2015).

For preference parameters, β=0.99, φ=1, and σ=1. Let θ=0.75, which implies the composite parameter κ=0.1717.

The persistence parameter (ρ_z) of the discount rate shock (z_t) is 0.5, and the standard deviation (σ_z) is 0.005. For the technology shock (a_t), the persistence parameter ρ_a=0.5, and the standard deviation $\sigma_t^a = 0.01$.

The response coefficients of the nominal interest rate to inflation (ϕ_π) and the output gap (ϕ_y) are 1.5 and 0.5/4. The persistence parameter (ρ_u) of the interest rate shock (u_t) is 0.5, and the standard deviation (σ_u) is 0.0025.

Appendix C. Evaluating the Volatility of Variables in Figure 7

This appendix illustrates how to evaluate the volatility of the two scenarios in Figure 7 statistically.

To eliminate occasional factors, we consider multiple fixed interest rates that fall in the intervals of $[3.99\%, 4.01\%]$ and $[1.99\%, 2.01\%]$. First, for each interval, select 10 interest rate values. Next, for each value, run 10 simulations when the nominal interest rate is fixed at the value. Third, for each simulation calculate deviations of model variables from their steady-state and initial values at the beginning of the fixed period. Then compute the mean of the $10 * 10$ deviations from steady-state and initial values, respectively.

The mean deviation of output is -4.7% (from steady state) and -1.2% (from its initial value) in the left column, compared with -2.0% (from ss) and -0.5% (from its initial value) in the right column. The mean deviation of inflation is -1.4% (from ss) and -3.7% (from its initial value) in the left column, compared with -0.6% (from ss) and -1.8% (from its initial value) in the right column. These results prove that output and inflation in the left column drop more than those in the right column in Figure 7.

The maximum deviation of output is -6.8% (from the steady state) and -3.4% (from its initial value) in the left column, lower than -3.5% (from ss) and -2.0% (from its initial value) in the right column; the maximum deviation of inflation is -4.8% (from ss) and -7.1% (from its initial value) in the left column, lower than -3.2% (from ss) and -4.3% (from its initial value) in the right column. These results also confirm the previous conclusion.

Appendix D. Calculating the Correlation in Figure 8

This appendix illustrates how to calculate the correlation between YoY output growth and the real interest rate in Figure 8.

For the correlation under the flexible interest rate, it converges around -0.28 with the extension of the simulation periods.

For the correlation under the fixed interest rate, like in Appendix C, we choose 10 fixed interest rates for each interval of $[3.99\%, 4.01\%]$ and $[-4.01\%, -3.99\%]$. Run 50 simulations for each interest rate, and calculate the correlation between YoY output growth and the real interest rate during the fixed period for each simulation. Finally, calculate the average of all $20 * 50$ correlations.