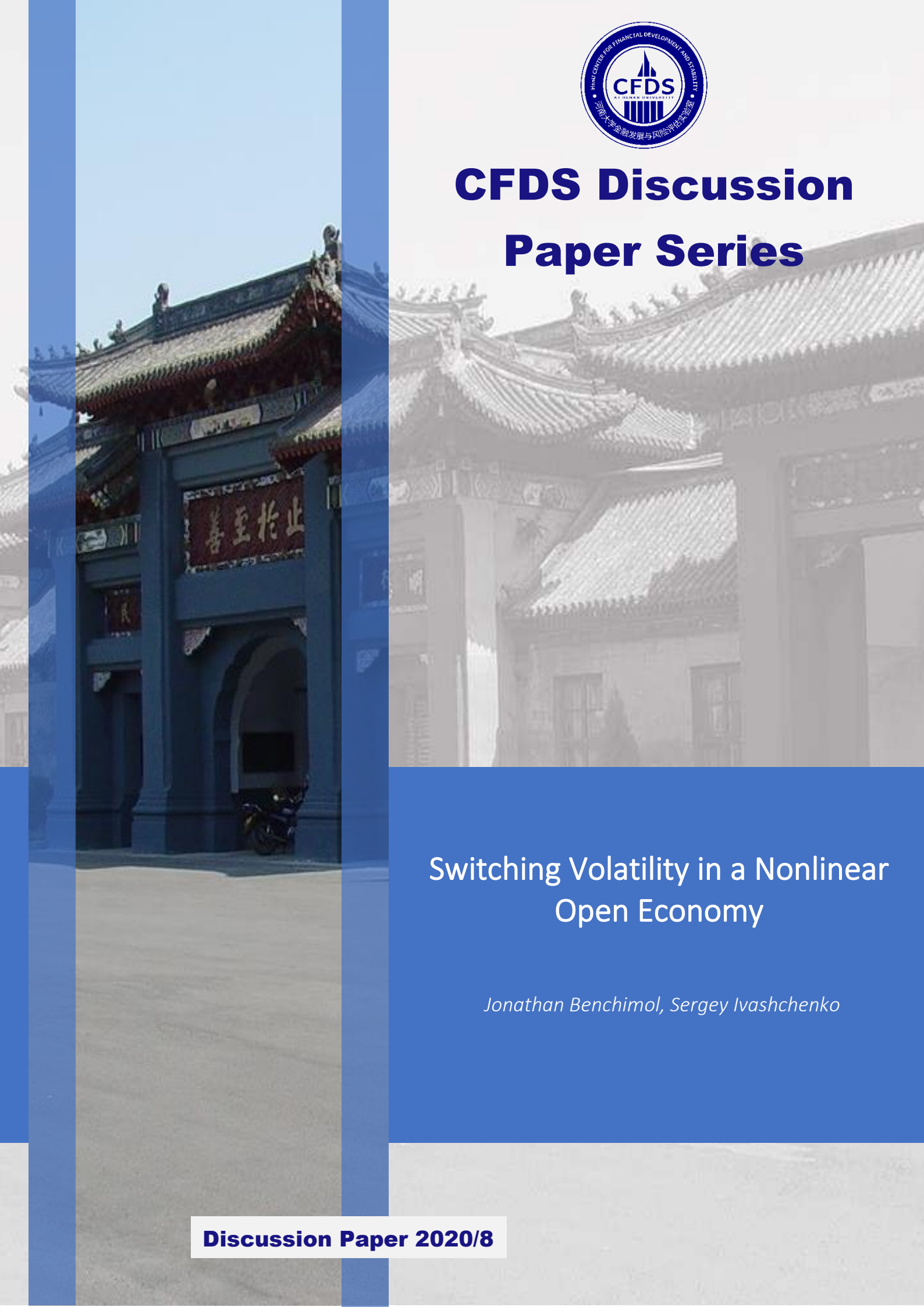

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Switching Volatility in a Nonlinear Open Economy

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Discussion Paper 2020/8

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Switching Volatility in a Nonlinear Open Economy*

Jonathan Benchimol† and Sergey Ivashchenko‡

Abstract

Uncertainty about an economy's regime can change drastically around a crisis. An imported crisis such as the global financial crisis in the euro area highlights the effect of foreign shocks. Estimating an open-economy nonlinear dynamic stochastic general equilibrium model for the euro area and the United States including Markov-switching volatility shocks, we show that these shocks were significant during the global financial crisis compared with periods of calm. We describe how US shocks from both the real economy and financial markets affected the euro area economy and how bond reallocation occurred between short- and long-term maturities during the global financial crisis. Importantly, the estimated nonlinearities when domestic and foreign financial markets influence the economy, should not be neglected. The nonlinear behavior of market-related variables highlights the importance of higher-order estimation for providing additional interpretations to policymakers.

Keywords: DSGE, Volatility Shocks, Markov Switching, Open Economy, Financial Crisis, Nonlinearities.

JEL Classification: C61, E32, F21, F41.

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1 Introduction

The widespread consensus in macroeconomics based on the linear new Keynesian model was shaken by the global financial crisis (GFC). Linear closed-economy dynamic stochastic general equilibrium (DSGE) models were not concerned with the sharp variance changes, economic structural breaks, and distribution shifts around the GFC. Consequently, regime-switching DSGE models have become the natural framework for analyzing macroeconomic dynamics (Maih, 2015).

An economic regime change could be related to a severe domestic or foreign financial crisis. The GFC started in the United States and affected the euro area (EA), thus changing the global economic environment for both economies. This switching process and analysis of such an international transition's volatility are not possible with the standard (linear) closed-economy DSGE models commonly used in the literature. For example, while classical DSGE models cannot reproduce switching volatility effects at all, linear Markov-switching DSGE (MSDSGE) models reproduce them only partially.

Indeed, linear DSGE models are useful for describing global macroeconomic stylized facts, but not all economic dynamics can be replicated (Smets and Wouters, 2003, 2007), even though central banks frequently use them to assist forecasting and monetary policy decisions as well as provide a *narrative* to the public (Edge and Gürkaynak, 2010). A nonlinear model estimated at higher-order solutions is thus essential for analyzing volatility shocks (Fernández-Villaverde et al., 2011), term structure (Rudebusch and Swanson, 2012), risk premia (Andreasen, 2012), and welfare dynamics (Garín et al., 2016).

In particular, higher-order approximations of DSGE models are crucial for determining whether changing (switching) volatility is a driving force behind business cycle fluctuations (Bloom, 2009). According to Markov processes, the volatility of several shocks can change over time. Furthermore, Markov-switching (MS) models provide tractable ways to study agents' expectation formation about changes in the economy, such as those occurring during a crisis (Foerster et al., 2016).

A vast body of the literature on dynamic open-economy models has emerged in the past two decades (Galí and Monacelli, 2005; Adolfson et al., 2007; Justiniano and Preston, 2010). However, analyses of the dynamic impacts resulting from regime-switching volatility changes in such a framework are scarce. Specifically, no study has used MSDSGE models with switching volatility shocks (SVSs). Based on the foregoing, we bridge this gap by considering the consequences of SVSs in a two-country MSDSGE model.

One way of influencing the variance of stochastic processes driving the economy necessitates third-order approximations with the usual perturbation method (Fernández-Villaverde and Rubio-Ramírez, 2013). Although our model is relatively simple, this method would involve including more than 30 state variables

and 10 autoregressive exogenous processes in the model, slowing the third-order approximation and model estimation. In addition, this approach suggests a slow drifting of volatility, whereas high levels of volatility switching are more often seen during crises. This characteristic is generally captured by MS processes in which a second-order approximation is required to analyze volatility shocks (Andreasen, 2010). For this purpose, we use nonlinear approximation algorithms and filters to estimate our MSDSGE models (Binning and Maih, 2015; Maih, 2015). However, for the various reasons presented in Appendix A, we develop and use a generalization of the quadratic Kalman filter applied to MSDSGE models.[1]

As domestic and foreign transmission channels were substantial during the GFC as well as in previous crises (King, 2012; Benchimol and Fourçans, 2017), two relevant transmission channels complete the model. Households can buy or sell domestic or foreign bonds in the long or short term and their money holdings increase their utility.

The model is estimated using the EA and US quarterly data compiled from 1995Q2 to 2015Q3 under three specifications: a baseline version without MS, a version allowing MS in technology only, and another more developed version allowing MS in three exogenous processes for each country, namely technology, home, and foreign monetary policy processes. To the best of our knowledge, this study is the first attempt to introduce long-term interest rates with embedded SVSs into a nonlinear open-economy DSGE model.

This exercise provides several interesting results and policy implications. First, we show and quantify that the average US and EA responses to shocks are different, especially around 2009Q1, which is also the case from the switching volatility point of view. These differences essentially come from the nonlinearities in economic dynamics, although our results are close to those obtained with linear open-economy DSGE models (Chin et al., 2015). Second, we demonstrate the consequences of SVSs on US and EA economic dynamics. SVSs produce a combination of short-term deflation and long-term inflation effects in line with Kiley (2014) but with some asymmetries between the two economies. We demonstrate that SVSs partially cause financial flows, showing that they significantly affect both the trade-off between short- and long-term bonds and consumption around the crisis. Third, we confirm that SVSs have a stronger impact on US monetary policy than on EA monetary policy. The latter result has several policy implications, such as monetary policy uncertainty switches.

Our results suggest that policymakers should use nonlinear models to address open-economy and market-related variables, which are subject to more nonlinear dynamics than standard closed-economy variables are. Comparing our models and estimations, we also show that considering a common technology and both domes-

[1] Appendix A presents the MS quadratic Kalman filter (MSQKF) we use.

tic and foreign monetary policy SVSs better describes the US and EA dynamics.

The remainder of this paper is organized as follows. Section 2 presents the model used for the estimation presented in Section 3. Section 4 presents the results and Section 5 interprets them. Section 6 concludes, and the Appendix presents additional results.

2 The model

Our generic model is a symmetric two-country model in which domestic (d) and foreign (f) households maximize their respective utilities subject to their budget constraints (Section 2.1), firms maximize their respective benefits (Section 2.2), and central banks follow their respective *ad-hoc* Taylor-type rules and budget constraints (Section 2.3). The model's equilibrium (Section 2.4) and stochastic structure (Section 2.5) are also presented in this section.

2.1 Households

For each country $i \in \{d, f\}$, we assume a representative infinitely lived household seeking to maximize

$$E_t \left[\sum_{t=0}^{\infty} \varepsilon_{i,t-1}^{u} U_{i,t} \right], \tag{1}$$

where $\varepsilon_{i,t-1}^{u} < 1$ is the exogenous process corresponding to households' country-specific intertemporal preferences,[2] and $U_{i,t}$ is households' country-specific intertemporal utility function, such as

$$U_{i,t} = \frac{\left(\hat{C}_{i,t} - h_i \hat{C}_{i,t-1}\right)^{1-\frac{1}{\sigma_{i,c}}}}{1-\frac{1}{\sigma_{i,c}}} + \varepsilon_{i,t}^{m} \frac{\left(\hat{M}_{i,t}/P_{i,t}\right)^{1-\frac{1}{\sigma_{i,m}}}}{1-\frac{1}{\sigma_{i,m}}} - \varepsilon_{i,t}^{l} \frac{L_{i,t}^{1+\frac{1}{\sigma_{i,l}}}}{1+\frac{1}{\sigma_{i,l}}} - \Psi_{i,t}, \tag{2}$$

where $\hat{C}_{i,t}$ is the detrended country-specific Dixit and Stiglitz (1977) aggregator of households' purchases of a continuum of differentiated goods produced by firms, $\hat{M}_{i,t}$ indicates the detrended country-specific end-of-period households' nominal money balances ($M_{i,t}/Z_t$), Z_t is the common level of technological progress,[3] $P_{i,t}$

[2] At time t, households know their intertemporal preferences for $t+1$ but have uncertainty about their preferences for the future. Hence, they know their preference multiplier for $t+1$. While they know $\varepsilon_{i,t}^{u}$ at time t, they do not know $\varepsilon_{i,t+1}^{u}$ at time t. Because utilities for $t+1$ should be multiplied by $\varepsilon_{i,t}^{u}$, current period utilities should be multiplied by $\varepsilon_{i,t-1}^{u}$.

[3] The existence of a common stochastic trend (common level of technology progress) requires stationary summands in the utility function. Consequently, the detrended consumption ($\hat{C}_{i,t} = C_{i,t}/Z_t$) and real money ($\hat{M}_{i,t}/P_{i,t}$) summands of this utility function satisfy the stationarity

is the country-specific Dixit and Stiglitz (1977) aggregated price index and $\Psi_{i,t}$ is the country-specific cost function described by Eq. 3. $\sigma_{i,c}$ is the country-specific intertemporal substitution elasticity of habit-adjusted consumption (i.e., inverse of the coefficient of relative risk aversion), $\sigma_{i,m}$ is the country-specific partial interest elasticity of money demand, and $\sigma_{i,l}$ is the country-specific Frisch elasticity of labor supply. $\varepsilon^m_{i,t}$ and $\varepsilon^l_{i,t}$ are the country-specific exogenous processes corresponding to real money holding (liquidity) preferences and the worked hours (disutility of labor) of households, respectively.

The country-specific household's cost function, $\Psi_{i,t}$, is defined by

$$\Psi_{i,t} = \frac{1}{2} \sum_{j \in \{sr,lr\}} \varphi_{i,d,j} \left(\frac{B_{i,d,j,t}}{P_{i,t} C_{i,t-1}} - \mu_{i,d,j} \right)^2 + \varphi_{i,f,j} \left(\frac{e_{i,t} B_{i,f,j,t}}{P_{i,t} C_{i,t-1}} - \mu_{i,f,j} \right)^2, \qquad (3)$$

where $\forall k \in \{d, f\}$ and $\forall j \in \{sr, lr\}$, $\varphi_{i,k,j}$ and $\mu_{i,k,j}$ are scale parameters related to the bonds' rigidity,[4] and $B_{i,k,j,t}$ represents the j-term k-bonds bought by

condition as in Adolfson et al. (2014). See, among others, Fagan et al. (2005), Schmitt-Grohé and Uribe (2011), and Diebold et al. (2017) for similar detrending. A stochastic trend with drift is suggested by the data—nonzero mean growth rate of macro-variables. Any DSGE model without trends is unrelated to real-world statistics and any approximation of a solution in initial terms—without removing trends—will not satisfy the Blanchard and Kahn (1980) conditions–explosive solution. Although the use of several trends is better (Schmitt-Grohé and Uribe, 2011), it requires a much more complicated model.

[4] When two agents with different intertemporal preferences trade the same security—especially bonds—credit-borrowing constraints are mandatory to avoid agents taking unrealistic positions. Thus, we add a quadratic portfolio adjustment rigidity for each type of bond position in the household's utility function, which produces smoothed restrictions. To simplify, we do not modulate such rigidity by restricting negative values. Although our approach is close to the portfolio adjustment costs *à la* Schmitt-Grohé and Uribe (2003) or price rigidity *à la* Rotemberg (1982), we assume preference costs in the utility function, while Schmitt-Grohé and Uribe (2003) assume real costs in the budget constraint. As it is more likely that households feel disutility from deviations in their financial position from the steady state, we do not assume that real goods are required to compensate for these deviations. Schmitt-Grohé and Uribe (2003) provide four methods to eliminate a unit root from an open-economy model. One comprises complete asset markets and identical discount factors for domestic and foreign households. The other specifications consider an exogenous foreign interest rate. As our model differentiates domestic and foreign households' discount factors and considers an endogenous foreign interest rate, these methods are not helpful. Our motivation for portfolio costs in the utility function is also technical. It allows us to exclude both the unit root and the cost from the resource constraint. We modify the utility portfolio adjustment costs' method to develop the model. Real portfolio adjustment costs should be considered as some component of GDP, which hardly corresponds to the national account system. By contrast, utility portfolio adjustment costs do not create such a problem. In the case of a first-order approximation at a deterministic steady state, these types of costs are equivalent. However, such a modification is necessary in the case of a higher-order approximation, while it does not affect the outcome or propagation mechanism concerning the original adjustment cost of Schmitt-Grohé and Uribe (2003).

households in country i in period t, where k represents the issuing country of the bond and j its maturity (i.e., short-term (sr) or long-term (lr) bonds). $e_{i,t}$ is the country-specific exchange rate relating to the number of domestic currency units available for one unit of foreign currency at time t (i.e., $e_{d,t} = 1/e_{f,t}$).

The market consists of domestic and foreign one-period short- and long-term bonds. Long-term bonds pay country-specific shares (S_i) of their current nominal value in each period.[5] In practice, S_i defines the bond duration (*average* time until cash flows are received).

Then, $\forall i \in \{d, f\}$, the country-specific households' budget constraint can be expressed as follows:

$$\begin{aligned} & P_{i,t}C_{i,t} + M_{i,t} + \sum_{j \in \{sr,lr\}} B_{i,i,j,t}Q_{d,j,t} + e_{i,t}B_{i,-i,j,t}Q_{-i,j,t} \\ & = W_{i,t}L_{i,t} + M_{i,t-1} + D_{i,t} \\ & + B_{i,i,sr,t-1} + B_{i,i,lr,t-1}\left(\left(1 - S_i\right)Q_{i,i,lr,t} + S_i\right) \\ & + e_{i,t}B_{i,-i,sr,t-1} + e_{i,t}B_{i,-i,lr,t-1}\left(\left(1 - S_{-i}\right)Q_{i,-i,lr,t} + S_{-i}\right), \end{aligned} \tag{4}$$

where index $-i$ denotes the other country (i.e., if $i = d$, then $-i = f$; if $i = f$, then $-i = d$) and $Q_{k,j,t} = \exp\left(-r_{k,j,t}\right)$ denotes the price of $r_{k,j,t}$, which is the country-specific (k) nominal interest rate at maturity j. $W_{i,t}$ is the country-specific wage index and $D_{i,t}$ represents the dividends paid by firms in country i at time t. The online appendix provides the optimality conditions.

Some DSGE models include a single variable for the lump-sum tax and dividends in the budget constraint (Schmitt-Grohé and Uribe, 2011), whereas others use two separate variables (Smets and Wouters, 2007). To simplify our model, we do not include a lump-sum tax and report only the dividends instead.

Money and the money demand shock do not influence the economy in the case of separable (additive) money in the utility function (Galí, 2015). However, the nonexistence of a lump-sum tax in our model that controls the bond position changes this mechanism. Our model has no such restrictive lump-sum taxation, which leads to the influence of money (and the money demand shock) on the economy.

2.2 Firms

The continuum of identical firms, in which each firm produces a differentiated good using identical technology, is represented by the following production function:

$$Y_{F,i,t}\left(j\right) = A_{i,t}L_{i,t}\left(j\right), \tag{5}$$

[5] A long-term bond with a nominal value of one domestic currency unit produces S_d units of the domestic currency in the first period, $S_d\left(1 - S_d\right)$ in the second period, $S_d\left(1 - S_d\right)^2$ in the third period, and so on. Because inflation-linked bonds are relatively rare and have lower liquidity in the United States and EA, we price bonds in nominal terms.

where $A_{i,t} = A_i Z_t$ is the country-specific level of technology, assumed to be common to all firms in country i and evolving exogenously over time, and A_i is a country-specific total factor productivity scale parameter.

As in Galí (2015), to simplify our analysis, we do not include the capital accumulation process in this model, which appears to play a minor role in the business cycle (Backus et al., 1992) , and assume constant returns to scale for simplification purposes.[6] The exogenous process Z_t introduces a stochastic trend into the model to explain the nonzero steady-state growth of the economy (Chaudourne et al., 2014; Diebold et al., 2017). Although alternative techniques to introduce a unit root exist (Schmitt-Grohé and Uribe, 2011), they complicate the model. For instance, Smets and Wouters (2007) reconstruct the deterministic component of the trend, which reduces the model accuracy.

All firms face an identical isoelastic demand schedule and take the country-specific aggregate price level, $P_{i,t}$, and aggregate consumption index, $C_{i,t}$, as given. Following Rotemberg (1982), our model features monopolistic competition and staggered price setting and assumes that a monopolistic firm faces a quadratic cost of adjusting nominal prices measured in terms of the final good given by

$$\frac{1}{P_{i,t} Z_t} E_t \left[\sum_{s=0}^{\infty} \frac{D_{i,t+s} - \varphi_{i,p} \left(\frac{P_{i,t+s}(j)}{\bar{P}_{s,i,t} P_{i,t+s-1}(j)} - 1 \right)^2 P_{i,t+s} Y_{i,t+s}}{\prod_{k=0}^{s-1} R_{i,t+k}} \right], \tag{6}$$

where $\bar{P}_{s,i,t} = \exp\left(v_i \overline{\pi_i} + (1 - v_i)\, \pi_{i,t+s-1}\right)$ represents the country-specific weighted average between country-specific steady-state inflation, $\overline{\pi_i}$, and country-specific previous inflation, $\pi_{i,t-1}$, in period t, where v_i is the country-specific weight and $\pi_{i,t} = \ln\left(P_{i,t}/P_{i,t-1}\right)$.

$P_{i,t}(j)$ is the price of goods j from firms in country i in period t, $R_{i,t} = \exp\left(r_{i,t}\right)$ is the short-term nominal interest rate, and $\varphi_{i,p} \geq 0$ is the degree of nominal price rigidity in country i. The country-specific adjustment cost, which accounts for the negative effects of price changes on the customer–firm relationship in country i, increases in magnitude with the size of the price change and with the overall scale of the country-specific economic activity $Y_{i,t}$.

In each period t, the firm's budget constraint requires

$$D_{i,t} + W_{i,t} L_{i,t} = P_{i,t}(j)\, Y_{i,t}(j), \tag{7}$$

[6] In this simple case, we also do not consider money in the production function. Several examples exploring this particular set-up are available in the literature (Benchimol, 2015; Gorton and He, 2016). Given the complexity of our model and empirical exercise, we assume long-term exogenous growth in a model without capital. Further research should analyze the benefits of capital as a factor of production to explain long-term growth.

where $Y_{F,i,t}(j)$ represents firms that manufacture goods j in country i in period t. Firms cannot make any investment (Eq. 7) and distribute all their benefits through dividends (Eq. 6).

The final consumption good is a constant elasticity of substitution composite of domestically produced and imported aggregates of intermediate goods that produces demand for firm output, such as

$$Y_{F,i,t+s}(j) = \omega_i Y_{i,t+s} \left(\frac{P_{i,t+s}}{P_{i,t+s}(j)} \right)^{\varepsilon^p_{i,t+s}} + (1-\omega_{-i}) Y_{-i,t} \left(\frac{e_{i,t+s} P_{-i,t}}{P_{i,t+s}(j)} \right)^{\varepsilon^p_{-i,t+s}}, \quad (8)$$

where the exogenous process $\varepsilon^p_{i,t+s}$ represents the country-specific price markup shock (elasticity of demand in country i), and the parameter ω_i defines a country-specific preference for local demand.

The aggregate country-specific price level also follows the usual constant elasticity of substitution aggregation, such as

$$P_{i,t}^{1-\varepsilon^p_{i,t}} = \omega_i P_{i,t}(j)^{\,1-\varepsilon^p_{i,t}} + (1-\omega_i)\left(e_{i,t} P_{-i,t}(j)\right)^{1-\varepsilon^p_{i,t}}, \quad (9)$$

where the local price index includes domestic and foreign prices as is usual in open-economy models.

2.3 Central bank

Central banks follow a Taylor (1993)-type rule, such as

$$R_{i,t} = \varepsilon^r_{i,t} R_{i,t-1}^{\rho_{i,r}} \hat{\pi}_{i,t}^{\rho_{i,\pi}(1-\rho_{i,r})} \hat{y}_{i,t}^{\rho_{i,y}(1-\rho_{i,r})} \hat{e}_{i,t}^{\rho_{i,e}(1-\rho_{i,r})}, \quad (10)$$

where $\varepsilon^r_{i,t}$ captures the country-specific monetary policy shocks, $\hat{\pi}_{i,t}$ is the country-specific inflation gap expressed as the ratio between country-specific CPI and its corresponding steady state, $\hat{y}_{i,t}$ is the country-specific output gap expressed as the ratio between country-specific output (normalized by technological progress) and its corresponding steady state, and $\hat{e}_{i,t}$ is the country-specific real exchange rate gap expressed as the ratio between the real exchange rate of country i and its corresponding steady state.

The parameter $\rho_{i,r}$ captures interest rate-decision smoothing, and $\rho_{i,\pi}$, $\rho_{i,y}$, and $\rho_{i,e}$ capture the weight placed by the monetary authority of country i on the inflation gap, output gap, and real exchange rate, respectively.

A standard budget constraint applies to the debt bought by central banks, such as

$$\frac{B_{i,g,t}}{R_{i,t}} = B_{i,g,t-1} + M_{i,t} - M_{i,t-1}, \quad (11)$$

where $B_{i,g,t}$ represents the country-specific nominal bonds bought by the local central bank in period t.

In our model, we assume that central banks can buy only short-term bonds, as was the case in the United States and EA before the GFC.

2.4 Equilibrium

In the equilibrium, country-specific demand consists merely of consumption, such as

$$Y_{i,t} = C_{i,t}, \tag{12}$$

and each bond should be bought, requiring that

$$B_{i,i,sr,t} + B_{-i,i,sr,t} + B_{i,g,t} = 0, \tag{13}$$

and

$$B_{i,i,lr,t} + B_{-i,i,lr,t} = 0. \tag{14}$$

The country-specific demand presented in Eq. 12, $Y_{i,t}$, is different from the country-specific supply presented in the production function (Eq. 5), $Y_{F,i,t}$. As in Berka et al. (2018) which also has only one source of demand, this simplification (Eq. 12) substantially decreases the number of variables, which is crucial for running a nonlinear estimation.

2.5 Stochastic structure

The exogenous processes we use are defined as $\forall i \in \{d, f\}$ and $\forall j \in \{u, m, l, p, r, y\}$,

$$\phi^j_{i,t} = \eta_{i,j}\phi^j_{i,t-1} + \left(1 - \eta_{i,j}\right)\bar{\eta}_{i,j} + \xi_{i,j,t}, \tag{15}$$

where the parameter $\bar{\eta}_{i,j}$ defines the country-specific steady state of exogenous process j, $\eta_{i,j}$ the country-specific autocorrelation level, and $\xi_{i,j,t}$ the country (i) shock-specific (j) white noise (zero-mean normal distribution).

The demand elasticity exogenous process is defined by $\phi^p_{i,t} = \varepsilon^p_{i,t}$, the intertemporal preference exogenous process by $\phi^u_{i,t} = \ln\left(\varepsilon^u_{i,t}/\varepsilon^u_{i,t-1}\right)$, technological progress by $\phi^y_t = \ln\left(Z_t/Z_{t-1}\right)$, and other exogenous processes by $\forall i \in \{d, f\}$ and $\forall j \in \{m, l, r\}$, $\phi^j_{i,t} = \ln\left(\varepsilon^j_{i,t}\right)$.

Appendix B summarizes the variables used in the model.

3 Methodology

In this section, we present the dataset used for the estimations (Section 3.1) as well as the estimation (Section 3.2) and computation of the nonlinear impulse response functions (IRFs) (Section 3.3).

3.1 Data

We estimate our model with quarterly EA (domestic) and US (foreign) data from 1995Q2 to 2015Q3 taken from the Organisation for Economic Co-operation and Development. In addition, we use the euro/dollar (EUR/USD) exchange rate from the European Central Bank (ECB) and Federal Reserve Bank of St. Louis (FRED) economic data for the exchange rate before the creation of the EA in 1999. The 11 observed variables are as follows: real gross domestic product (GDP) growth rate (EA and US), GDP deflator (EA and US), ratio of domestic demand to GDP (EA and US), 3-month interbank rate (EA and US), 10-year interest rate (EA and US), and EUR/USD growth rate.

With five country-specific shocks and one joint total factor productivity shock, the number of shocks is equal to the number of observed variables. Our model and empirical investigation include the long-term interest rate, allowing us to capture long-term bond demand/supply effects through their interest rates in both countries. We also capture monetary aggregate dynamics and negative interest rates. The use of the 3-month interbank rate from the Organisation for Economic Co-operation and Development database makes the zero lower bound problem less critical, as it becomes negative for the European Monetary Union in several periods. Consequently, although we do not explicitly model unconventional monetary policies, our data highlight some unconventional monetary policy effects.

3.2 Estimation

Our switching (two-regime) model is estimated in three ways with maximum likelihood techniques. First, we estimate a baseline version of our model without SVSs (i.e., without switching). As the productivity shock remains the main source of uncertainty in the business cycle (Bloom et al., 2018), another version is estimated by considering only one SVS in Z_t (hereafter, 1SVS). A third version considers both the productivity and the monetary policy SVSs: $\varepsilon^r_{d,t}$, $\varepsilon^r_{f,t}$, and Z_t (hereafter, 3SVS). The 3SVS model aims to capture the volatility regime switches during the GFC in both the United States and the EA, as suggested by Mavromatis (2018). Monetary policy and productivity shocks are the main driving forces of business cycles. Additional SVSs are feasible in theory; however, in practice, they require significant additional computing resources and may not change the results or make the model more realistic.

The model solution approximation is computed with the efficient second-order perturbation method developed by Maih (2015). We use the MSQKF described in Appendix A, which is an extension of the QKF for the MS case (Ivashchenko, 2014). The switching volatility and second-order approximation features constitute the nonlinearities of our models. We use the first four quarters as a presample

of our three estimations and jackknife bootstrapping for robustness purposes.[7] The estimation results of these three models in Appendix C show that the 3SVS model, which includes switching volatility in the technology and monetary policy shocks, is the best model to explain current and forecasted aggregate and individual (observable) dynamics.

The share of steady-state inflation indexation (υ_i) differs across regions as well as in the different versions of the model. The coefficient for the United States is close to that of Smets and Wouters (2007). The version without switching has a larger share of steady-state inflation indexation. The other models could produce lower estimated values of the υ_i parameter, which are close to the 1SVS result for the EA, and even smaller for Canada, which is close to the EA results in the version without switching (Justiniano and Preston, 2010). The share of steady-state inflation indexation for the EA is much smaller. The 3SVS version produces the closest values of the corresponding parameters. Thus, volatility switching might influence inflation persistence, of which the share of past inflation indexation $(1 - \upsilon_i)$ is one of the key elements.

For the model with variance switching under multiple exogenous shocks, regime 2 has higher variance of Z_t. However, in this case, several variances in the second state are smaller.

Fig. 1 presents the filtered values of regime 1 probabilities and three selected exogenous processes ($\varepsilon^p_{d,t}$, $\varepsilon^p_{f,t}$, and Z_t). This figure shows $Prob\,(r_t = 1)$ conditional on the data probability, where $Prob\,(r_t = 1)$ corresponds to the probability of being in regime 1 in period t.

Only moderate differences exist between the filtered values of the exogenous processes. In addition, the differences in state probabilities are linked to the state of the 1SVS model, whereas the state probabilities of the 3SVS model are more reliable. The latter correspond to the actual main crises that occurred during the sample period. The difference between the filtered values of the exogenous processes is generally smaller before the GFC, whereas it is larger a few years after the beginning of the GFC. Economic driving forces are generally unaffected by SVSs, except at certain points in time, especially during crises. This is also the case when monetary policy shocks are considered.

[7] Our table of observations has 11 columns (observables) and 82 rows (periods). We randomly discard four observations from this table and perform maximum likelihood estimation. We repeat this process more than 100 times and receive a robust variance estimation. Our methodology (i.e., jackknife bootstrapping) is different from prefiltering, as it does not use the likelihood values corresponding to the first four quarters for all the variables. Jackknife bootstrapping suggests discarding four observations randomly and combining the variable and period.

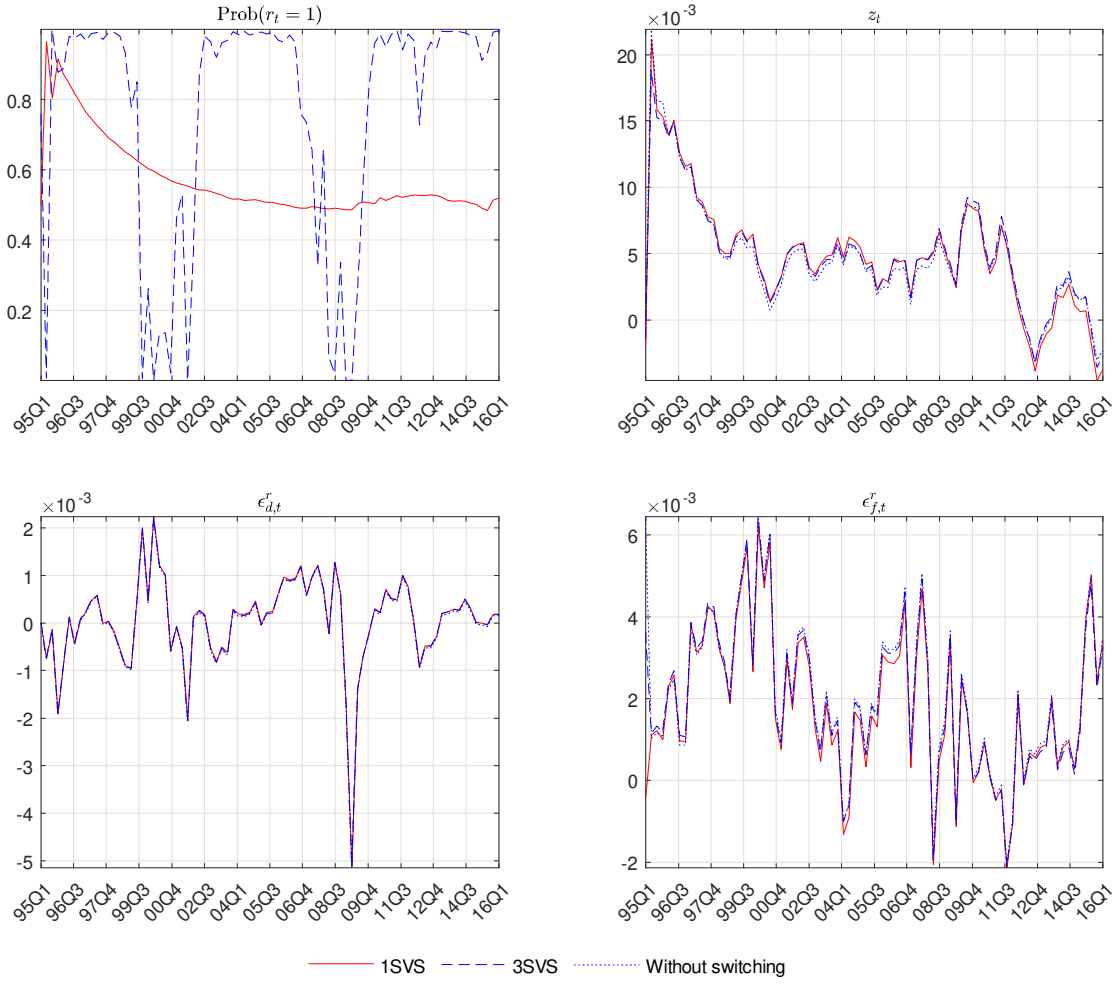

Figure 1: Regime probability, technology (Z_t), US ($\varepsilon^p_{d,t}$), and EA ($\varepsilon^p_{f,t}$) price markup shocks.

3.3 Impulse response functions

To analyze the response of the variables to economic shocks, we compute for each variable its IRF to each shock. The standard definition, such as presented in Dynare (Adjemian et al., 2011), defines the IRF as the expected difference between the trajectory with one shock in a single period one standard deviation higher and the usual trajectory. More precisely, we express this as

$$IRF_t(x,\xi) = E\left[x_t|\xi_1 \sim N\left(\sigma\left(\xi\right),\sigma\left(\xi\right)\right)\right] - E\left[x_t|\xi_1 \sim N\left(0,\sigma\left(\xi_1\right)\right)\right], \qquad (16)$$

where x_t is the value of the variable of interest for which the IRF is computed in period t, ξ_1 is the shock of interest that deviates in period 1, $\sigma(.)$ is the standard error operator, $E[.]$ is the expectation operator, and N is the normal law.

We generalize this definition in the nonlinear case by making the magnitude and sign of the shock more important. Such a generalization requires the introduction of the parameter s in Eq. 16 to determine the number of standard deviations in the shock, such as

$$IRF_{t,s}(x,\xi) = \frac{E[x_t|\xi_1 \sim N(\sigma(\xi_1)s, \sigma(\xi_1))] - E[x_t|\xi_1 \sim N(0, \sigma(\xi_1))]}{s}. \quad (17)$$

In addition, we compute the IRFs conditional on the state variables' vector X_t to show the differences between the IRFs at different states of the world, such as

$$IRF_{t,s}(x_t,\xi|X_0) = \frac{E[x_t|\xi_1 \sim N(\sigma(\xi_1)s, \sigma(\xi_1)); X_0] - E[x_t|\xi_1 \sim N(0, \sigma(\xi_1)); X_0]}{s}, \quad (18)$$

where X_0 is a vector of the state variables before the shock.

The IRF for the switching shock is

$$IRF_t(x, v_0, v_1) = E[x_t|r_0 = v_0; r_1 = v_1] - E[x_t|r_0 = v_0], \quad (19)$$

where r_t is the regime variable at time t, and v_0 and v_1 are the switching values of the regime of interest.

To compute the expectations, we use a simulation with the same exogenous shocks for both parts of the IRF equation. We use 50,000 draws for averaging and 100 presample draws for the unconditional IRF.[8]

4 Results

In this section, we present the responses of our model after an SVS (Section 4.1) and a monetary policy shock (Section 4.2). Further, we present and analyze some nonlinearities (Section 4.3). The other results are available upon request. Appendix C presents additional performance measures showing the advantages of the volatility switching (i.e., 3SVS) model over the other models.

4.1 Switching volatility shock

Fig. 2 presents the IRFs of the SVSs from states 1 to 2 (with higher volatility for Z_t) for the 1SVS model. We compute the unconditional IRF and plot the mean IRF and +/- two standard deviations (std) of the IRF.

Fig. 2 shows that the regime probability effect disappears without strong persistence (around 10 periods). However, the effect on the model's variables is much

[8] We consider the steady state as the initial point and we draw the trajectory for 100 periods. The shock occurs in period 101, and we repeat this 50,000 times.

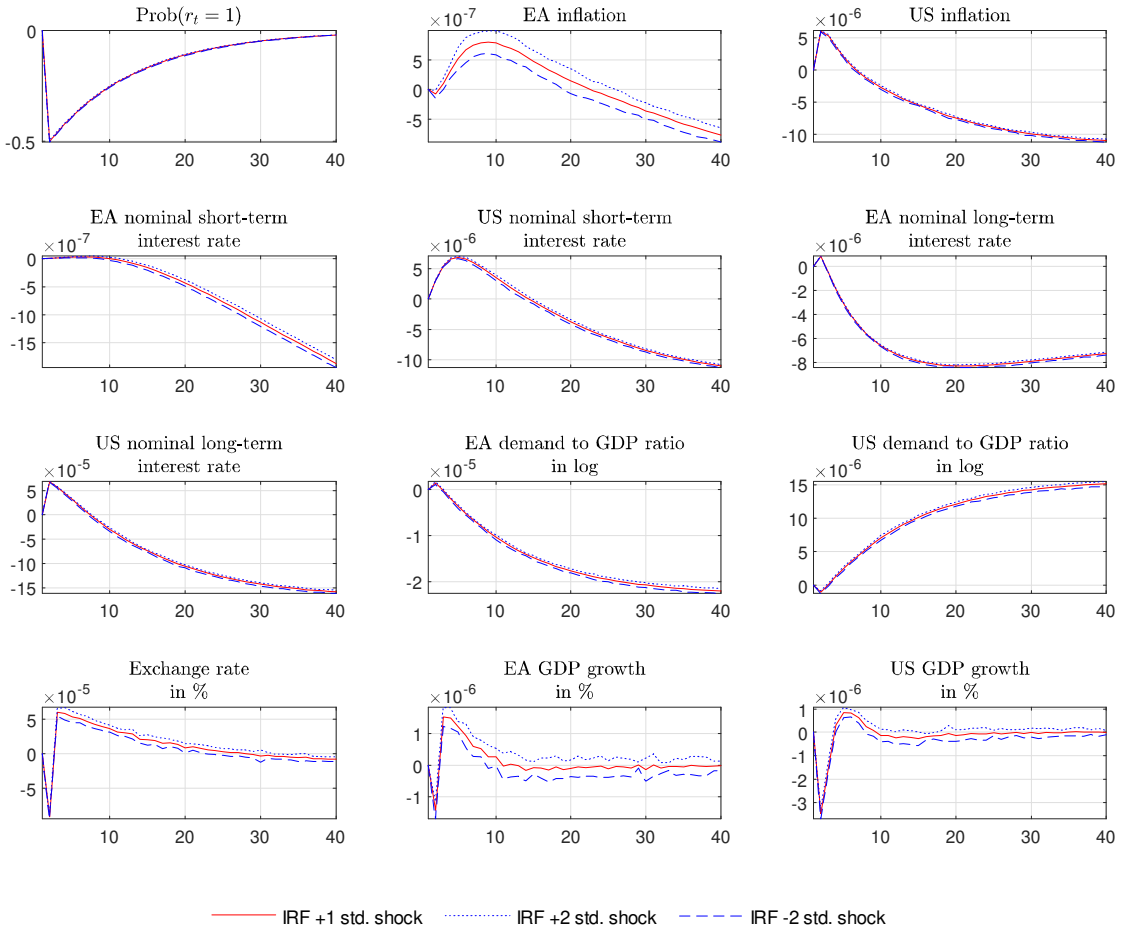

Figure 2: Unconditional IRFs to an SVS to regime 2 (1SVS).

more persistent and differs by region. Following an SVS, inflation increases in the two regions during the first periods, involving an increase in the US short-term nominal interest rate, while the EA's short-term nominal interest rate remains stable. The picture changes drastically in later periods when the long- and short-term interest rates in the United States and EA's both decrease with the inflation rates. Only GDP growth and the exchange rate are stabilized after several periods.

The US long-term rate decreases more smoothly a few quarters after the shock compared with the EA long-term nominal interest rate. This difference can be explained by the different durations of the long-term bonds in the EA ($s_d = 0.6$) and United States ($s_f = 0.06$).

In addition, monetary policy weights, by generating different short-term interest rates, could explain this phenomenon. The United States has a stronger response to inflation and a smaller smoothing coefficient than the EA. Consequently, the US short-term nominal interest rate decreases with inflation and in-

creases later, while that for the EA increases slightly. This difference in monetary policy produces fluctuations in the exchange rate and ratio of domestic demand to GDP.

Fig. 3 provides a more robust picture than Fig. 2.

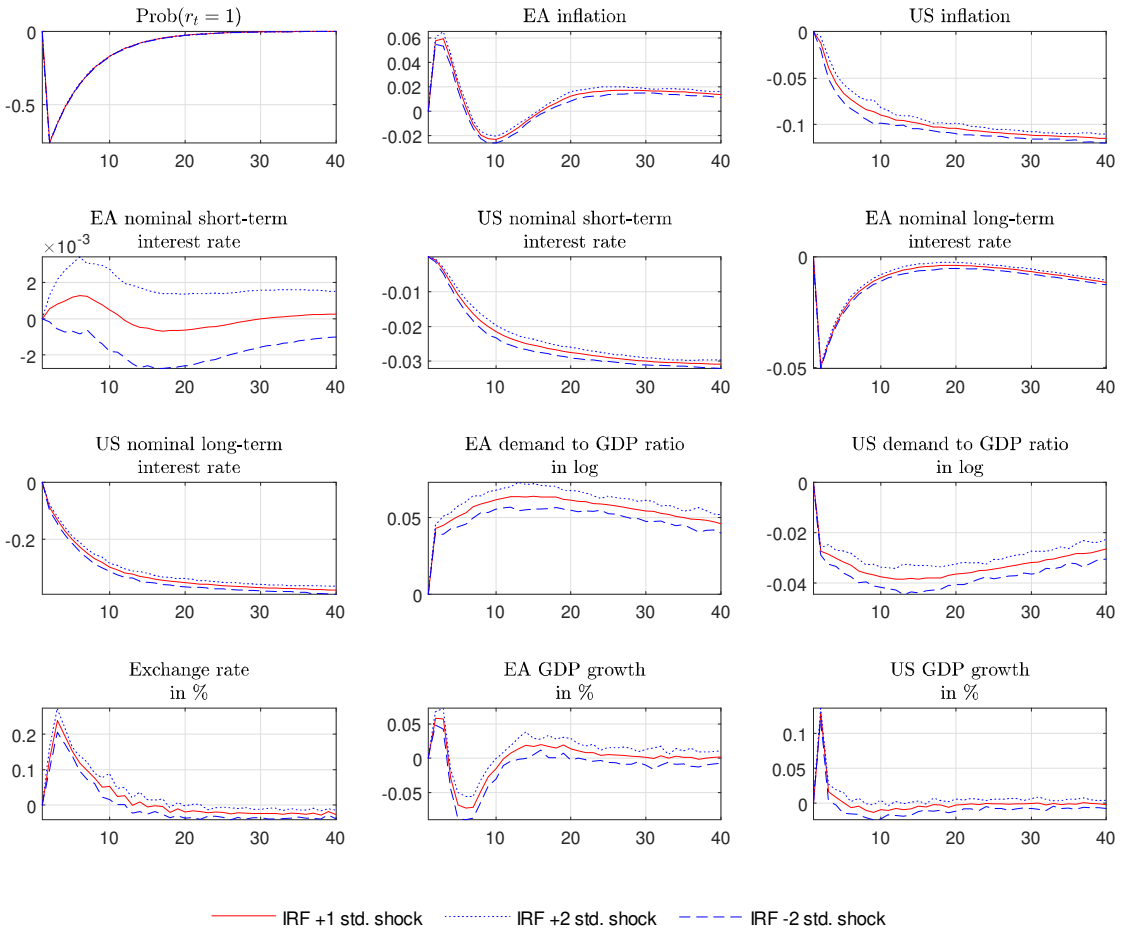

Figure 3: Unconditional IRFs to an SVS to regime 2 (3SVS).

Indeed, the 1SVS model suggests only a few differences between regimes (the standard deviations are close), implying a small effect on the economy of switching, which explains the low values obtained in Fig. 2. However, the 3SVS model suggests much larger differences and a substantial impact of switching shocks on the economy.

Fig. 3 highlights that SVSs affect US inflation and nominal interest rates in both the short and long terms, while the impact on the EA economy is less significant. Such SVSs durably influence US long-term interest rates, whereas this is not the case for the EA's long-term interest rates.

Uncertainty around the EA's short-term nominal interest rate, measured as the gap between -2 std and +2 std around the IRF, is stronger than that around the US short-term nominal interest rates.

In addition, the demand-to-GDP ratios of the two regions display substantial uncertainty, showing that the SVSs in the monetary policy shocks of the two regions have important economic implications.

In Fig. 3, the economy switches to regime 2, which means a substantial increase in the volatility of both foreign and domestic monetary policy shocks and a decrease in total factor productivity shock volatility. Higher uncertainty means higher interest rates. However, the central bank controls interest rates, buys bonds, and prints money that leads to higher inflation. As the economy is open, domestic changes are substantial, and foreign households buy more domestic bonds. Foreign households work more and sell more goods to the domestic country. Moreover, foreign investment in the domestic market makes foreign currency cheaper. Thus, foreign households increase investments and hold more money. As this effect is powerful, foreign inflation decreases, leading to lower foreign interest rates.

The average effect of unconditional SVSs might differ from that of conditional IRFs. For example, Fig. 4 compares unconditional IRFs with conditional IRFs for 2009Q1 and 2003Q4. Here, we use the filtered values of the variable vector for the corresponding dates as the condition (the initial point for a draw).

These IRFs are different in several aspects. Indeed, the regime probability IRF differs in the configurations shown in Fig. 4, where we compare the *best* expansion (2003Q4) with the *worst* recession (2009Q1) periods. This could be because the economy is in regime 2 when the shock occurs in the case of the unconditional IRF, while the conditional IRF could be in regime 1 before the shock.

Further, the 3SVS model highlights the significant differences between the crises as well as between the United States and EA (Fig. 5).

These differences are more reliable than in the 1SVS model. For instance, the EA short-run nominal interest rate was not similarly affected by the switching during good and bad times (e.g., the subprime crises) and their corresponding SVSs. Furthermore, significant differences are observed for the ratio of domestic demand to GDP in both regions.

An SVS has a stronger impact on the EA's demand-to-GDP ratio than on that of the United States, at least after the dot-com crisis. In addition, Fig. 5 shows that the EA consumes more, while the United States consumes less. At the same time, US inflation and interest rates decrease slightly more than the unconditional points and the EA.

The influence of the SVS is significant. For instance, in the long run, the US long-term interest rate change caused by an SVS is about 0.4% over ten years (Fig. 5). The EA demand to GDP changes of about 0.05% for ten years after an SVS,

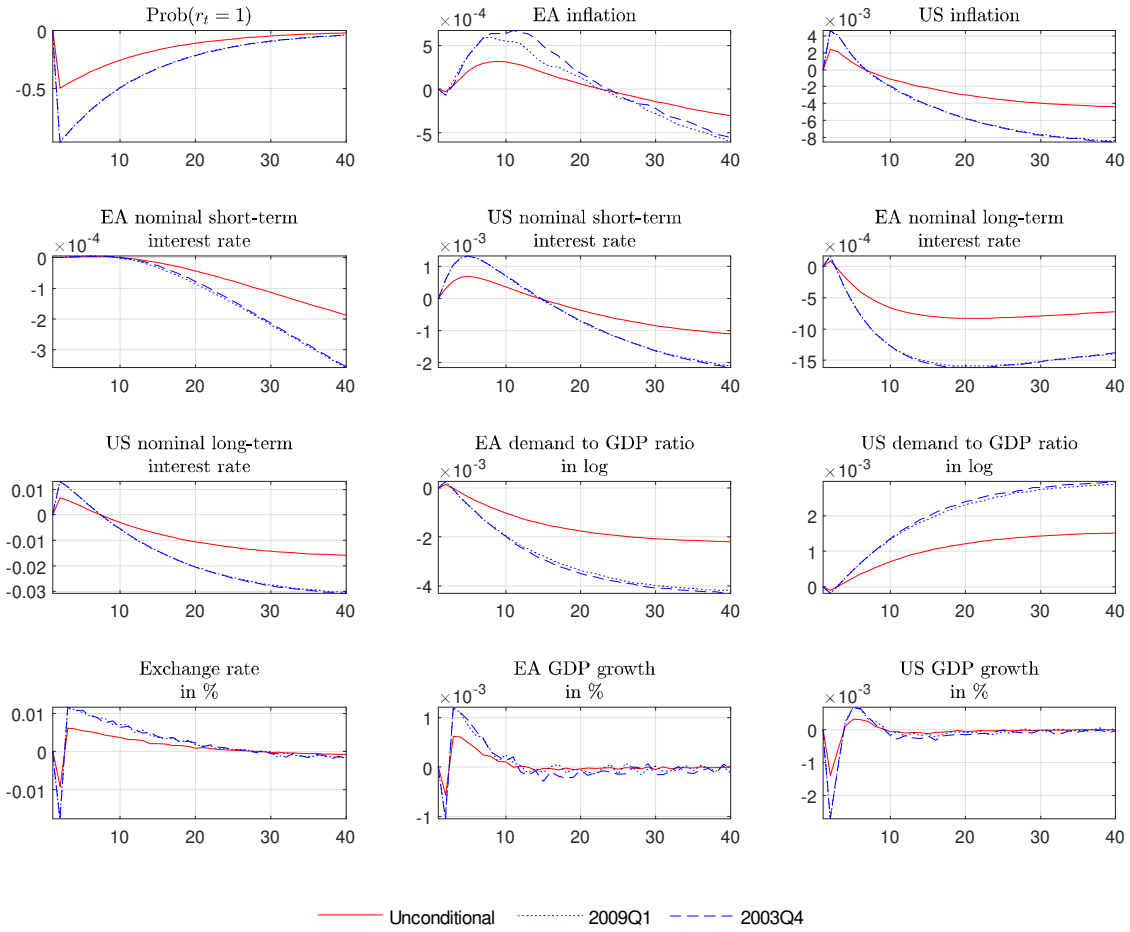

Figure 4: Conditional and unconditional IRFs to an SVS to regime 2 (1SVS).

which indicates a 0.5% cumulative change of trade (in terms of EA GDP). In the short run, the US GDP growth change resulting from the SVS is about 0.15%.

Fig. 6 compares the consequences of regime switching for both models. As expected, the IRFs are significantly different, mainly due to the switch of multiple variances. The response of the 3SVS model is less monotonic and the magnitudes of the IRFs are different for most of the economic variables.

Fig. 6 shows that the switch of multiple variances significantly affects the exchange rate as well as US long-run nominal interest rates, while short- and long-term nominal interest rates in the EA are less affected. However, the EA's demand-to-GDP ratio is more affected than the US ratio.

The 3SVS model captures several dynamics that a 1SVS model without switching cannot, such as the decreasing short-term nominal interest rates in the United States and oscillating inflation in the EA.

Fig. 7 shows that the 1SVS model influences the financial variables, uncondi-

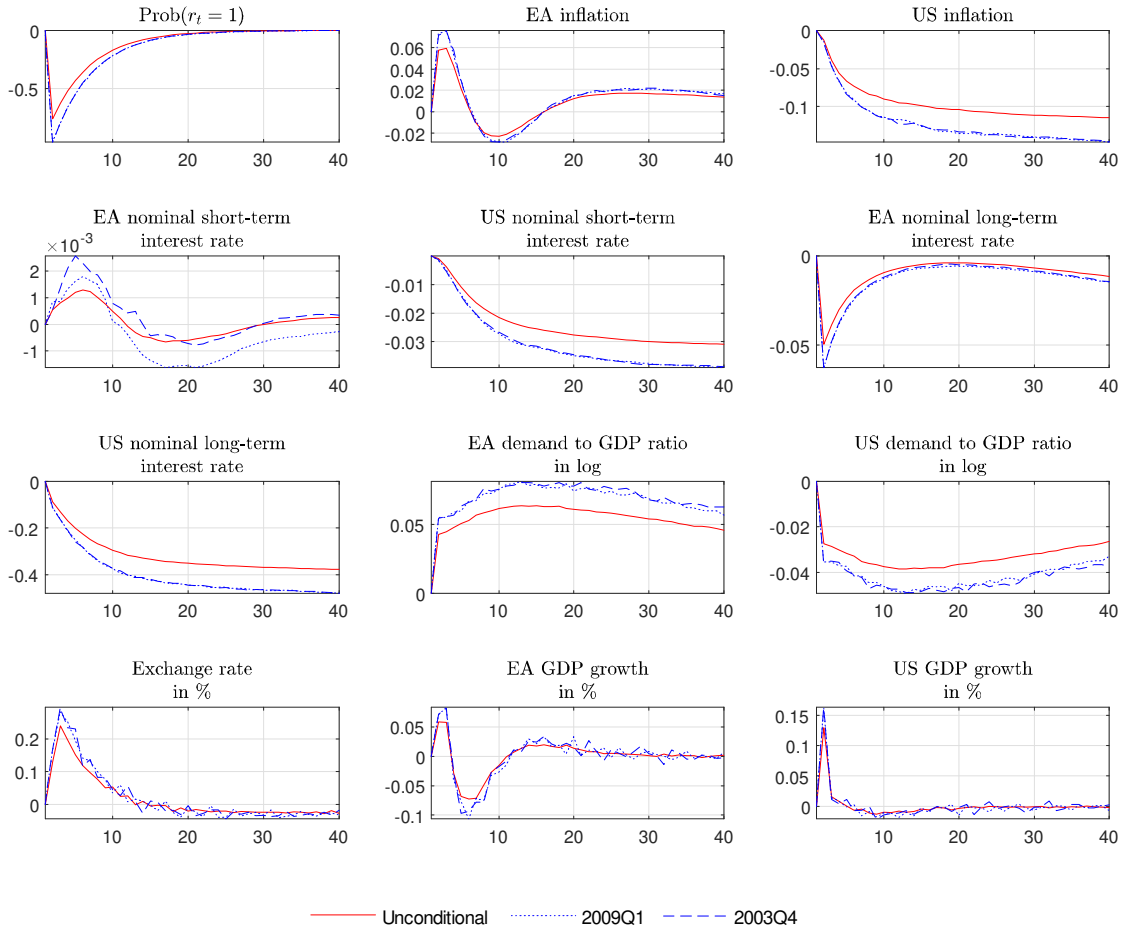

Figure 5: Conditional and unconditional IRFs to an SVS to regime 2 (3SVS).

tionally and conditionally, compared with the reference dates.

Although the magnitude of these IRFs is relatively low, some conclusions can be drawn. The 1SVS responses are clearly different during the subprime crisis and after the dot-com crisis, unconditional on time (Fig. 7). An SVS increases the US bonds bought by EA and US households as well as the EA bonds bought by the ECB (in the short run). Following such an SVS, the exchange rate, the EA's short- and long-term bonds bought by EA households and the Federal Reserve, and money held by US households all decrease.

The main problem in this scenario is that it assumes that the aftermaths of the dot-com and subprime crises are similar, at least in terms of the IRFs and the impact of an SVS on the financial variables. However, this was not the case; indeed, the financial transmission channels during these two crises were fundamentally different.

Fig. 8 shows a more coherent picture with significant and reliable differences

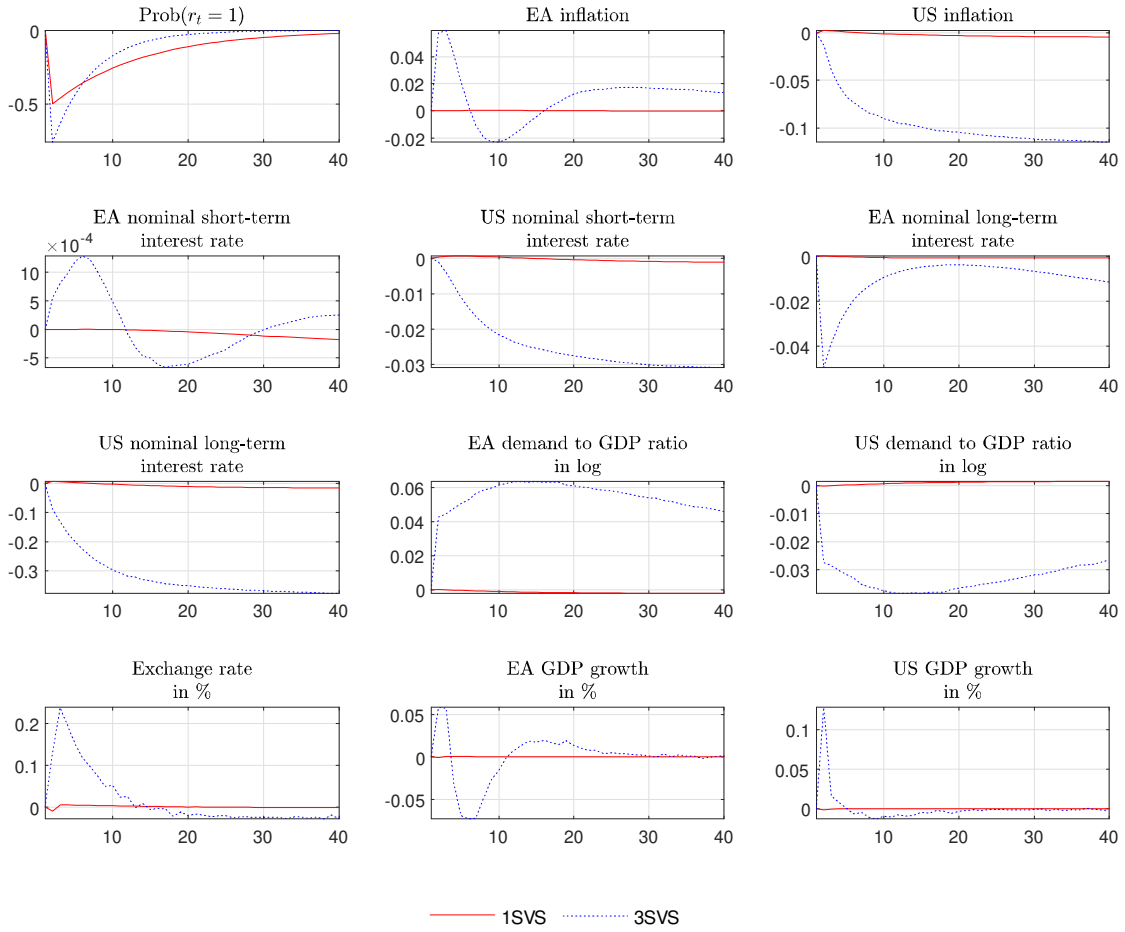

Figure 6: Unconditional IRFs to an SVS (to regime 2) for the 1SVS and 3SVS models.

in the IRF after the dot-com crisis and during the GFC.

Indeed, the 3SVS model during the GFC increased the US bonds bought by US households and EA bonds bought by the ECB, while this was not the case after the dot-com crisis or unconditionally. Such shocks also decreased the exchange rate and money held by EA households in all cases, while the money held by US households increased.

Fig. 8 shows that the response of US short-term bonds is due to an increase in the Federal Reserve's bond position, while other agents decrease their bond position. In the EA, the picture is different: the ECB slightly increases its bond position, and both European and US households decrease their EA long-term bond positions.

Then, because US and EA households are selling their US bonds, the construction of our model suggests that the Federal Reserve must buy them after the

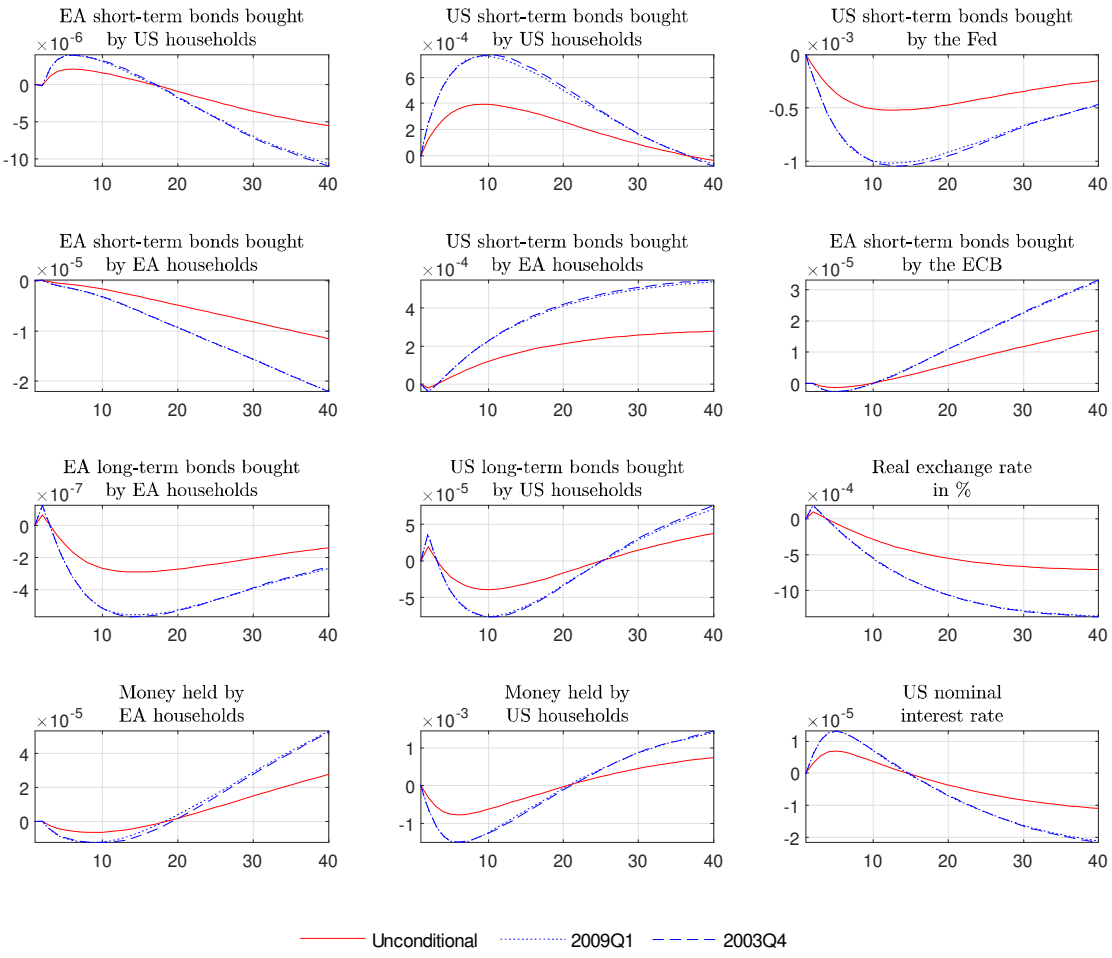

Figure 7: Conditional and unconditional IRFs to an SVS (to regime 2) for the financial variables (1SVS)

dot-com crisis. Such a result is close to the reality of the past decade.

Moreover, Fig. 7 shows that following such SVSs, both regions' households hold more money after several periods and sell EA long-term bonds. This result is a direct consequence of the increase in the short-term EA bond position and consumption. US households increase their overall bond position and money holdings, such that euros return to the EA and US dollars return to the United States.

Another interesting result lies in the differences between the 1SVS and 3SVS models. The 1SVS model (Fig. 7) hardly discriminates between the two conditional IRFs (2003Q4 and 2009Q1), while the 3SVS model (Fig. 8) differentiates between these two dates, which are economically (and financially) substantially different. Consequently, the 3SVS model could match the stylized financial facts better than the 1SVS model (and a fortiori compared with the baseline model

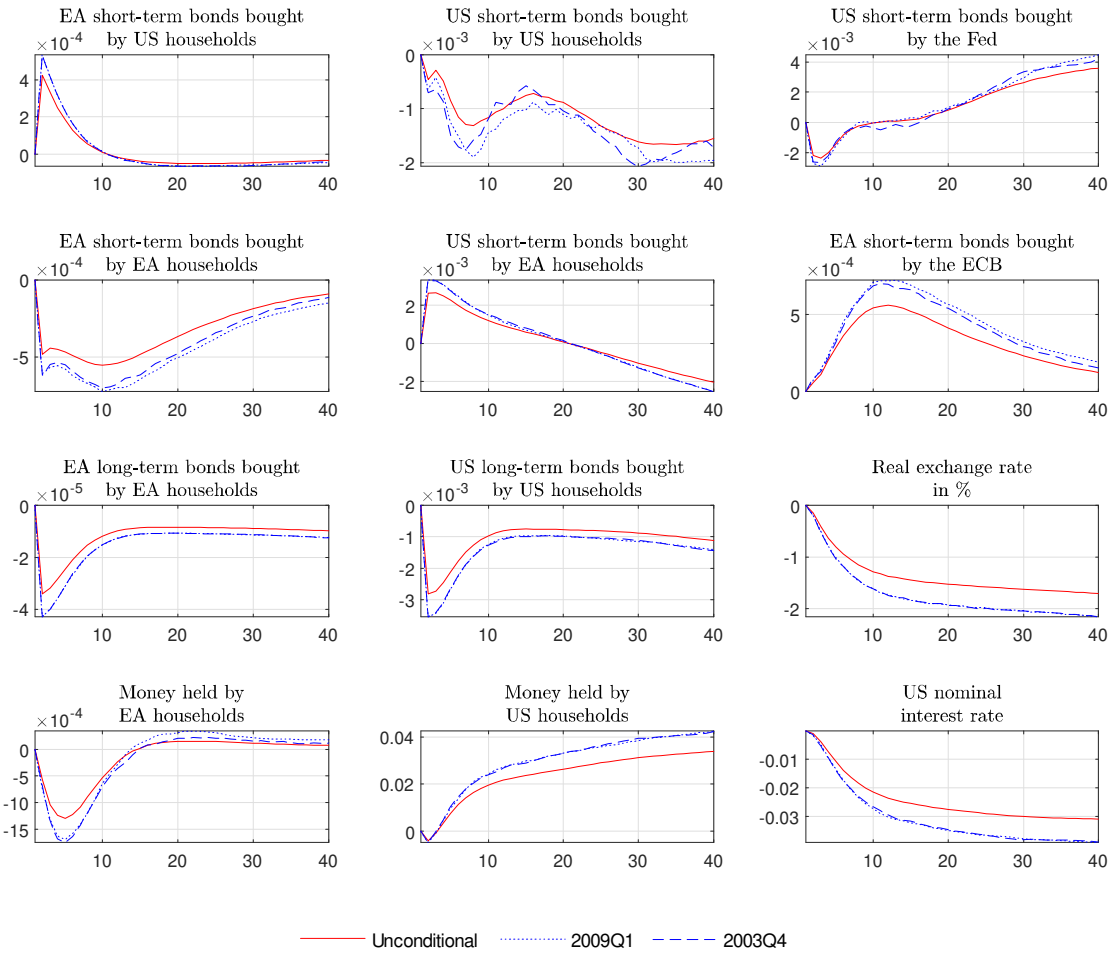

Figure 8: Conditional and unconditional IRFs to an SVS (to regime 2) for the financial variables (3SVS)

without switching).

In terms of the IRF levels, the 3SVS model brings about higher volatility to the responses of the economic variables, especially for the exchange rate, money holdings, and bond quantities. Volatility shocks were essential drivers of the GFC and, as we see hereafter, nonlinearities also affect economic dynamics.

Fig. 8 demonstrates the increasing real exchange rate difference of about 0.5% over 10 years. Such differences between conditional and unconditional IRFs show how nonlinearities are significant.[9] In the long run, an SVS leads to a change of about 2% in the real exchange rate. The SVS effect is very persistent with a substantial consequence, in that the difference between conditional and unconditional IRFs for the real exchange rate exceeds 0.2% over more than eight years

[9] See Section 4.3 for an analysis of nonlinearities.

(Fig. 8). The importance is also related to the duration of effect. For instance, if EA exports and imports represent about 53% of GDP,[10] the cumulative effect of a 0.2% change in exchange rates over eight years would lead to a flow of money representing 0.85% of yearly GDP (direct influence[11]).

4.2 Monetary policy shock

Fig. 9 shows the consequences of an EA monetary policy shock for each model. The responses are similar except that the US long-term interest rate is lower under the 3SVS model, while the price of long-term bonds is higher.

An EA monetary policy shock leads to higher inflation in Fig. 9. Hence, the real interest rate increases, leading to a lower money position and a higher bonds position. The government budget means that it creates additional income for households. This means higher consumption, which increases imports. Importation growth then leads to a cheaper national currency, and thereby inflation growth and domestic production growth.

However, the responses of a US monetary policy shock differ depending on the model (Fig. 10), especially for the demand-to-GDP ratio, long-term interest rates, and GDP growth in the first quarters. US inflation responses are more pronounced in the 3SVS model than in the model without SVSs.

In addition, EA and US growth rates are significantly different in the first quarters, showing that the model without switching allows more variability to US and EA growth in the first periods, with different signs at some points in time.

A foreign monetary policy shock leads to lower inflation in Fig. 10. The central bank places significant weight on inflation. Lower inflation expectations lead to lower inflation and interest rates, which then motivates households to increase money and decrease bonds. This produces an additional cash flow that is spent on consumption. Additional demand leads to higher imports. This makes the national currency relatively cheap and domestic production rises to some extent.

Interestingly, long-term interest rates have different responses in the United States and EA. While the US long-term nominal interest rate decreases sharply in the 3SVS model, the decrease in the EA long-term nominal interest rate is less pronounced. Without switching, the US long-term nominal interest rate decreases less than in the 3SVS model, while the EA long-term nominal interest rate increases more than in the 3SVS model. Thus, SVSs could provide relevant information for monetary policy decisions.

[10] The EA national accounts show that the share of exports is 28.2% of GDP in 2018. The share of imports is 24.7% of GDP during the same period. The exports and imports represented 53% of GDP.

[11] The exchange rate influenced both export and import payments leading to a total effect would be $0.002 \times 8 \times 0.53 = 0.85\%$.

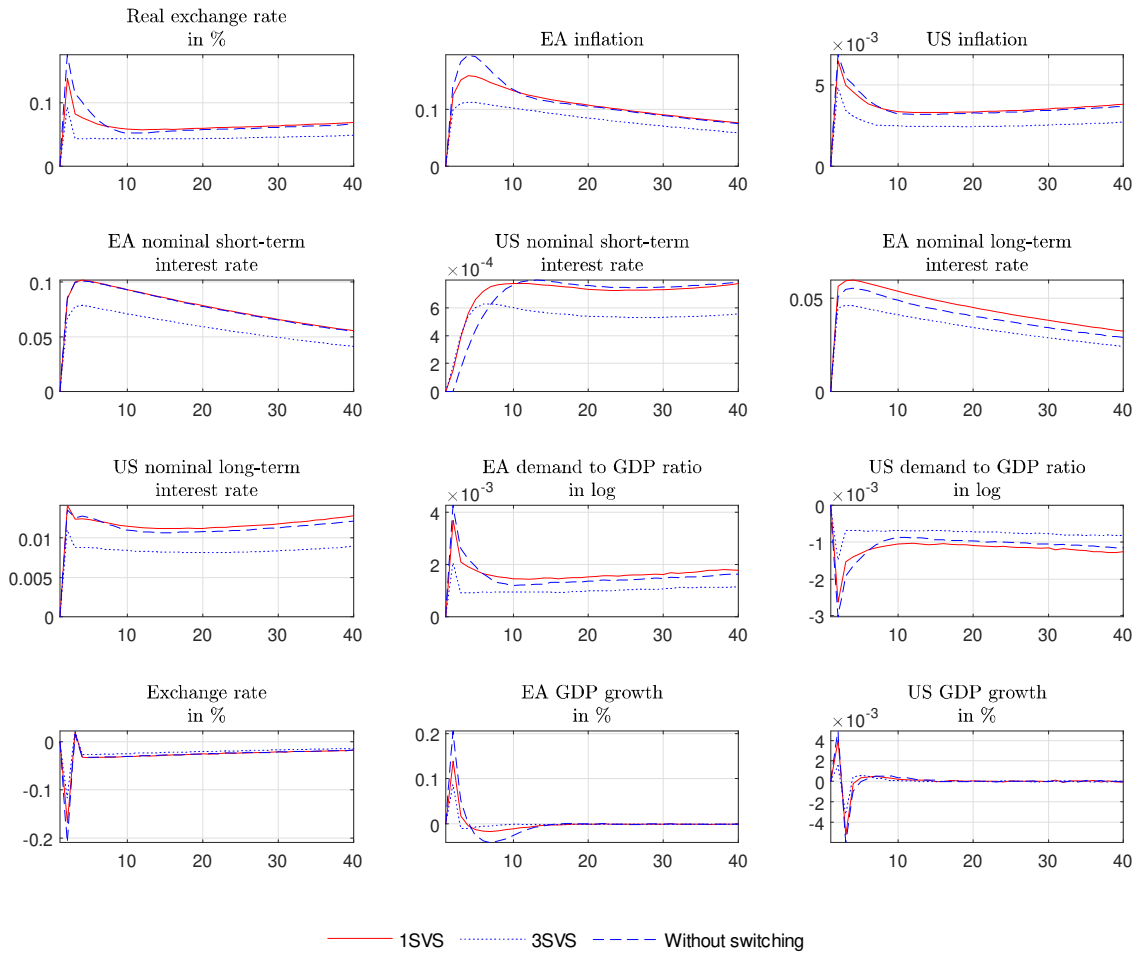

Figure 9: Unconditional IRFs to a positive EA monetary policy shock (one standard deviation).

In line with the stylized facts, a symmetric monetary policy shock does not have similar consequences if it is in the EA or the United States.

4.3 Nonlinearities

The previous IRF figures considered only a one standard deviation positive shock. However, in a nonlinear world, responses are also nonlinear. How should these nonlinearities be quantified? Fig. 11 to Fig. 14 present the unconditional IRFs after monetary policy shocks of different magnitudes to assess the importance of these nonlinearities.

Fig. 11 presents the IRFs after an EA monetary policy shock according to the 1SVS model.

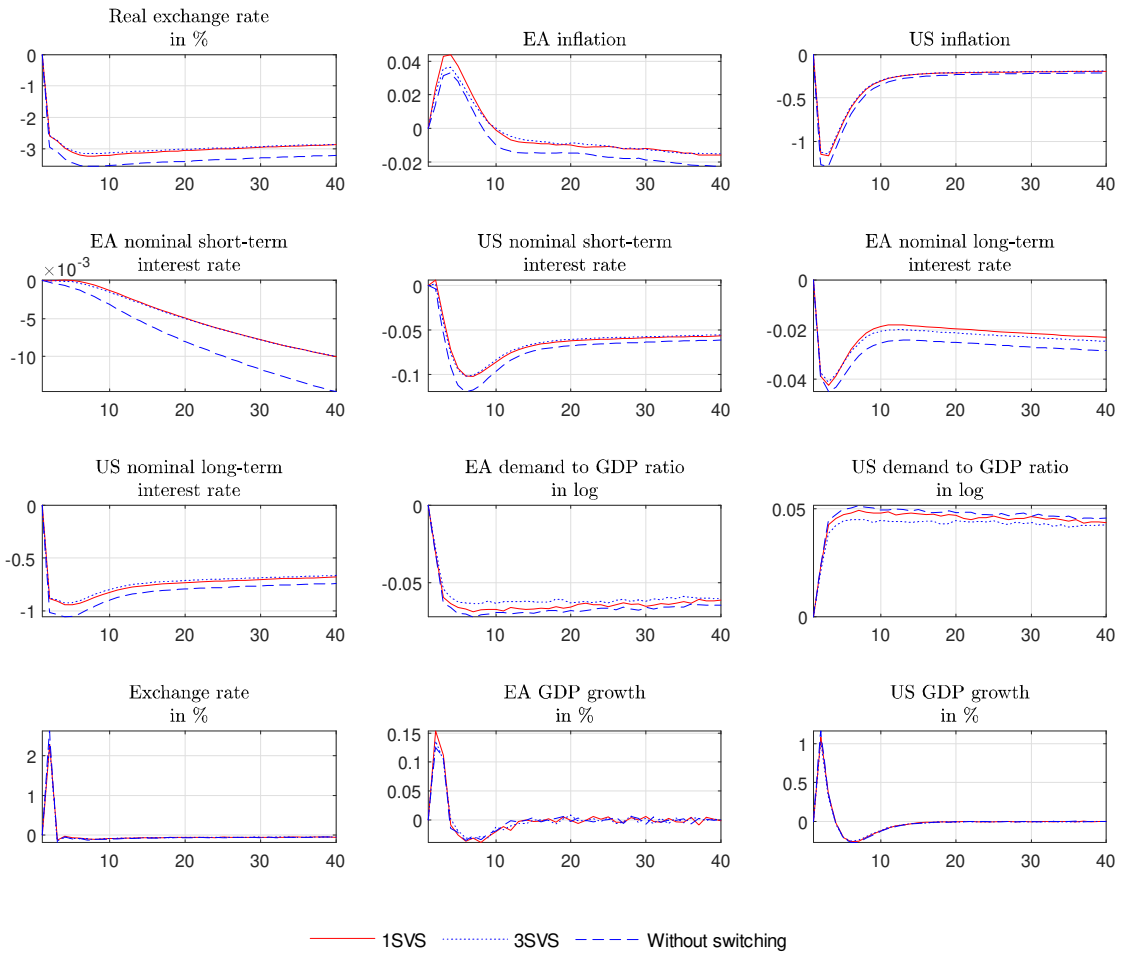

Figure 10: Unconditional IRFs to a positive US monetary policy shock (one standard deviation).

While +1 std and +3 std are similar, one crucial nonlinearity resides in -3 std, which is also similar to that for positive shocks. This nonlinearity is easily understandable mathematically (power 2), avoiding a symmetric response, which is standard in DSGE models' IRFs linearized at the first order.

However, this negative EA monetary policy shock (-3 std) has a lower response than the other positive shock, even though the direction is similar. Nonlinearities could lower the efficiency of monetary policy shocks, which is an important result for monetary authorities using simple linear models to assess economic situations and take monetary policy decisions.

Further, nonlinearities are more visible in the economy, and such a picture, as presented in Fig. 11, has a shallow impact (the scale is always between 10^{-3} and 10^{-5}).

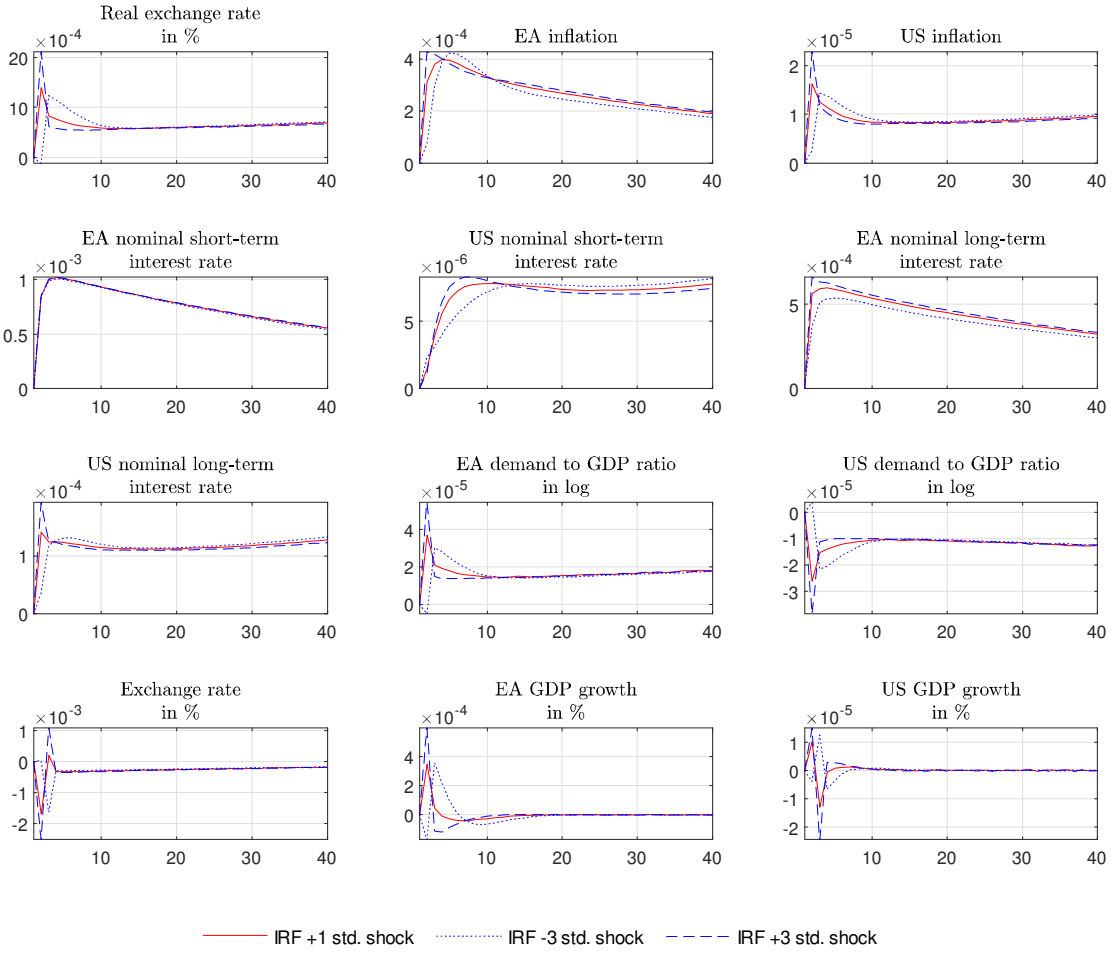

Figure 11: Unconditional IRFs to EA monetary policy shocks of different magnitudes (1SVS). std. stands for standard deviation.

Fig. 12 presents the IRFs after an EA monetary policy shock according to the 3SVS model.

Unlike Fig. 11, Fig. 12 presents the magnitudes of higher nonlinearities, and these results are more in line with those in the literature (An and Schorfheide, 2007), especially for exchange rates (Altavilla and De Grauwe, 2010).

Furthermore, nonlinearities influence EA inflation uncertainty in an interesting way. While the negative monetary policy shock (-3 std) has an important impact on EA inflation in the first periods, it exceeds +1 (+3 std) after several periods, highlighting the nonstandard perspective allowed by nonlinearities.

Moreover, Fig. 13 highlights an important result for policymakers. Responses to a US monetary policy shock have different nonlinearities than responses to an EA monetary policy shock (Fig. 11). EA inflation and the demand-to-GDP ratios

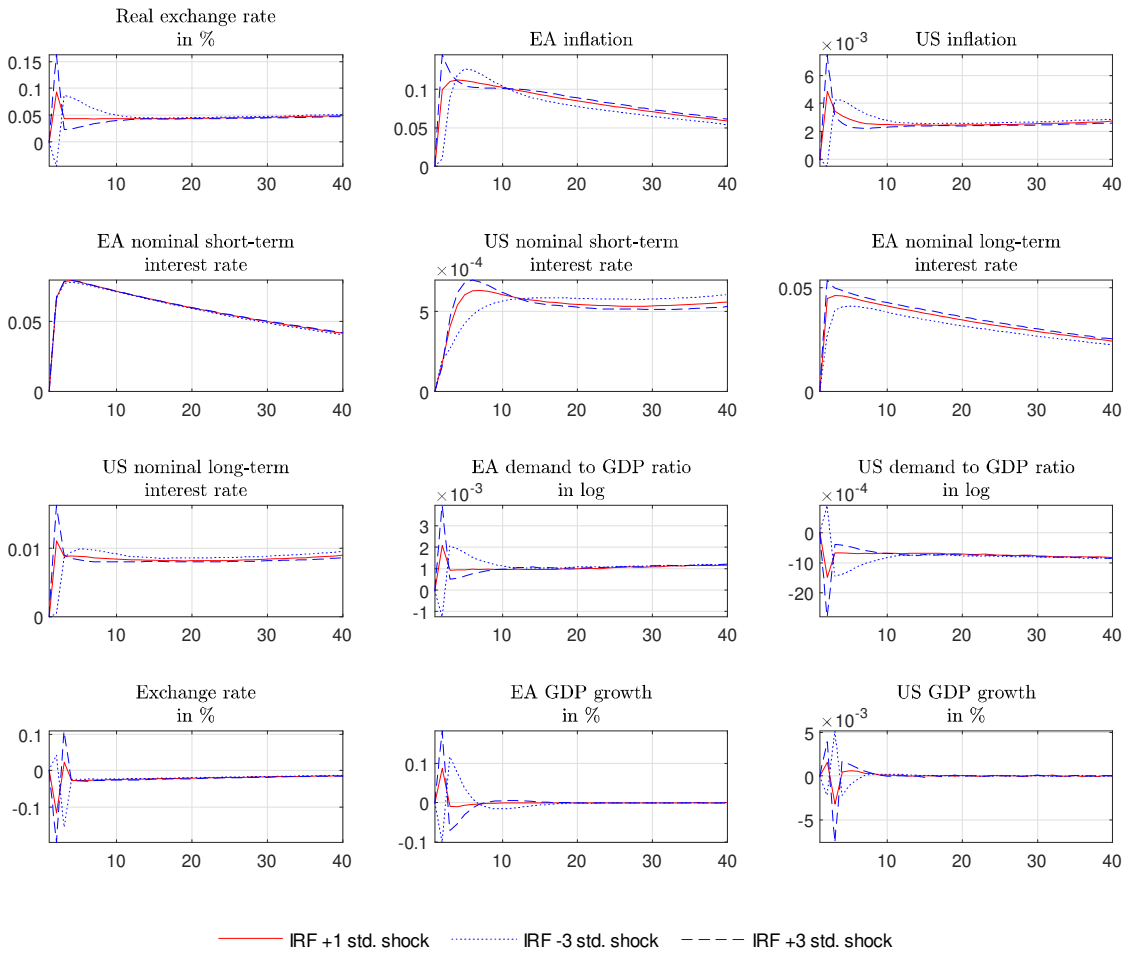

Figure 12: Unconditional IRFs to EA monetary policy shocks of different magnitudes (3SVS). std. stands for standard deviation.

of both the United States and the EA behave almost nonlinearly following a US monetary policy shock (at least in the first periods). In these cases, -3 std and +3 std are asymmetric, whereas we find small nonlinearities (asymmetries) in the previous case (EA monetary policy shock).

Interestingly, +1 std and +3 std US monetary policy shocks do not have the same consequences for EA growth (Fig. 13). This finding shows that nonlinearities help explain why strong monetary policy reactions do not have the same consequences as small monetary policy reactions. The same comment applies to the US demand-to-GDP ratio.

Thus, policymakers should analyze economic decisions, including their own, through the spectrum of nonlinear models to optimize the magnitude of their monetary policy reaction function.

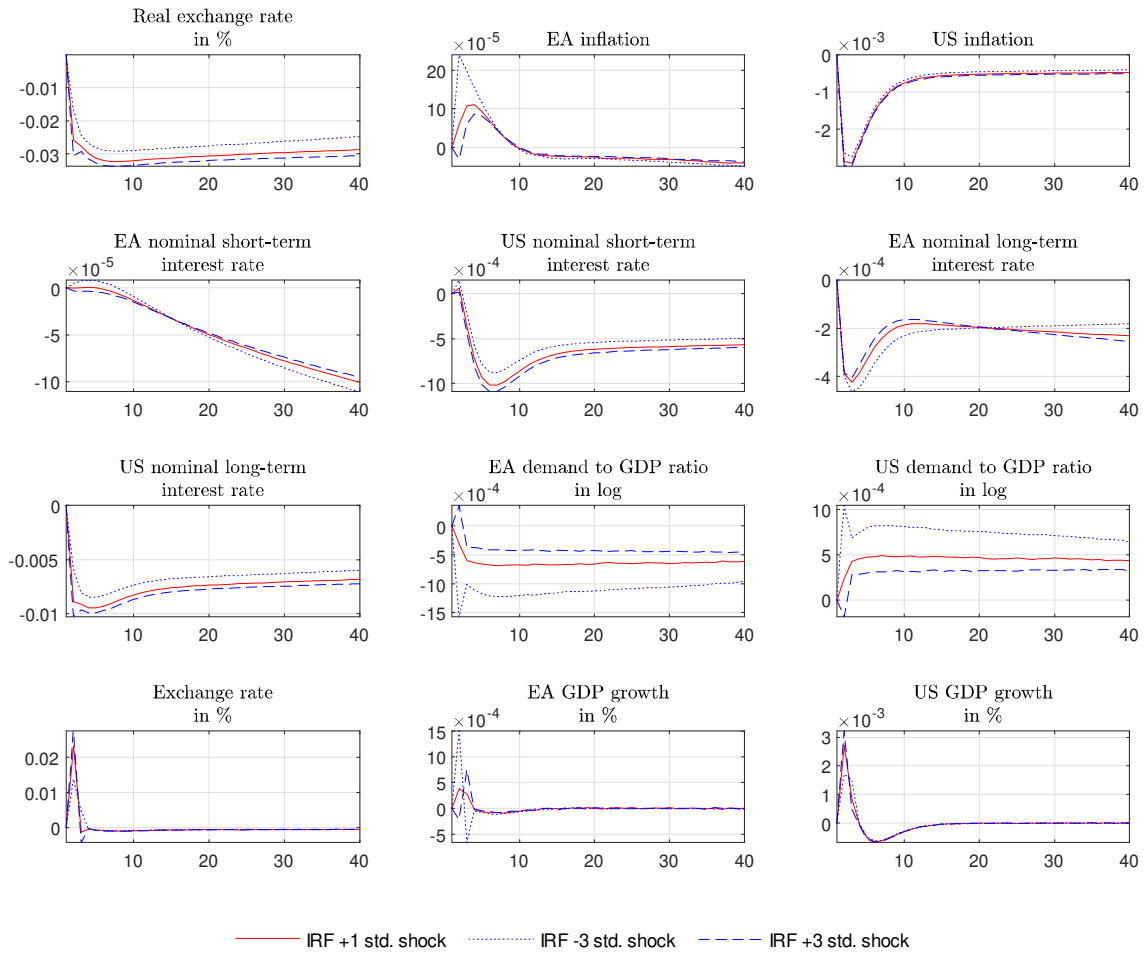

Figure 13: Unconditional IRFs to US monetary policy shocks of different magnitudes (1SVS). std. stands for standard deviation.

Fig. 14 presents the IRFs after a US monetary policy shock according to the 3SVS model.

Nonlinearities are present in the case of a US monetary policy shock (Fig. 14). EA inflation displays the same phenomenon as presented previously (Fig. 12), again denoting strong differences between the linear and nonlinear models.

Fig. 14 confirms that the persistent difference in the demand-to-GDP ratio depends on the shock's magnitude and sign. This difference is larger than 0.05% over ten years, meaning the cumulative change of trade equals 0.5% of GDP. The short-term difference in EA GDP growth rates exceeds 0.5% due to the shift in growth peak timing. More significant is the response of US GDP growth of about 1% after a US monetary policy shock of different magnitude.

Moreover, the role of SVSs is significant, especially concerning demand-to-

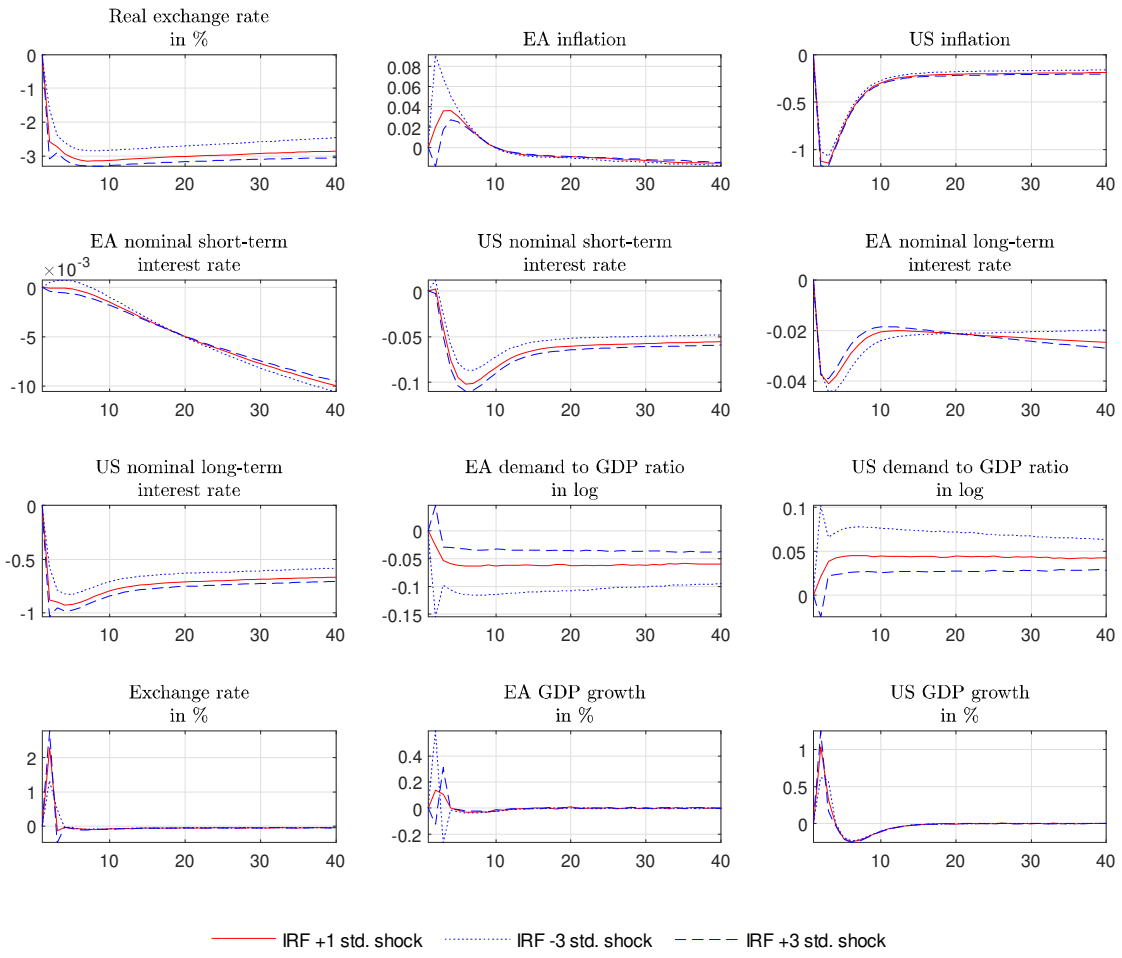

Figure 14: Unconditional IRFs to US monetary policy shocks of different magnitudes (3SVS). std. stands for standard deviation.

GDP ratios and exchange rates. Overall, the nonlinearities and SVSs on monetary policy shocks affect not only the magnitudes of the considered dynamics but also the actual dynamics as well as their orders.

Such a result is fundamental for policymakers and economists willing to model economies around a crisis. Open-economy models are suitable for such nonlinearities (Altavilla and De Grauwe, 2010) and masking nonlinearities using linear models to analyze such economies could lead to inadequate economic interpretations and policy decisions.

5 Interpretation

Section 2 presents an original model featuring households, firms, and the central banks of two economies, with households able to buy domestic or foreign short-term bonds. Following Kiley (2014), we show that the short-term nominal interest rate has a more substantial effect on the overall economy than the long-term nominal interest rate and that both short- and long-term interest rates are key determinants of consumption. However, our results also highlight that the EA's long-term interest rates comove strongly with US long-term rates rather than with short-term rates (Chin et al., 2015). This result is confirmed with the 3SVS model (Fig. 9) in which the US long-term nominal interest rate reacts more strongly to an EA short-term nominal interest rate shock.

Following Chin et al. (2015), we find that US disturbances influence EA economies markedly (Fig. 13). These results are confirmed by the variance decompositions of the variables with respect to the shocks (Appendix D) and distance correlations between the variables (online appendix).

In addition, we find that US money shocks affect the EA real variables in the long run as well as the financial markets in the EA and United States (Appendix D). We therefore extend the literature by highlighting new transmission channels of money compared with other studies using linear closed-economy DSGE models with money (Benchimol and Fourçans, 2012, 2017; Benchimol and Qureshi, 2020). Unlike this body of the literature, we show the role of money in the economy without assuming nonseparability between money and consumption (Benchimol, 2016), or a cash-in-advance constraint (Feenstra, 1986) or money in the production function (Benchimol, 2015). The money holdings from households and central banks needed to buy bonds involve such a role of money (Eq. 11). Although the additive separable utility function (Eq. 2) excludes real money balances from the IS curve (Jones and Stracca, 2008), money has a role through the money-in-the-utility function and households' budget constraints because of the *direct effect* (Eq. 3) highlighted by Andrés et al. (2009).

An interesting result on inflation's variance decomposition is that the price markup shock (demand elasticity shock) plays a critical role in the EA, while this shock explains only a small share of the inflation dynamics in the United States, illustrating how EA and US economies behave differently during crises. In the long run, this is explained by the strength of price markup shocks explaining domestic as well as foreign wage dynamics. This smaller effect of the price markup shock on the inflation rate in the short run can be caused by nonlinear dynamics, which are missing from standard closed-economy models.

Another result relates to the intertemporal preferences shock, showing that it has minor short-term explanatory power, whereas it becomes one of the most important shocks for explaining some variable dynamics in the long run (Appendix

D), such as the part of inflation dynamics not explained by the price markup shock. This fact is essential for models with domestic and foreign preference shocks. The absence of such a shock in the literature on open-economy DSGE models could conceal additional dynamics that could complete the economic scenarios developed by policymakers, such as on foreign and domestic bonds or private consumption.

Hence, our nonlinear open-economy DSGE model with several SVSs allows us to enrich the dynamics of interest rate markets for different maturities (Section 4.1). Fig. 2 shows that the US response to inflation is stronger than that of the EA. How can one conciliate this with the stabilization objectives of the Federal Reserve and ECB? The official objective of the Federal Reserve is to react to both inflation and output growth or unemployment, while the ECB's is to mainly react to inflation. However, these objectives differ from the concrete reaction to these variables. First, the existence of an additional component in the Federal Reserve's objectives does not mean a lower response to US inflation. We have shown that the Federal Reserve responds substantially more (in absolute values) to the output gap and exchange rate than the ECB, which could compensate for its response to inflation. As an active central bank, this stronger response to economic changes leads to faster stabilizing effects with more significant interest rate fluctuations compared with the ECB. The ECB's smaller responses smooth interest rates during a more extended (stabilization) period. Both monetary policies correspond to the official objectives but have significant differences in the preferences across the components of these objectives. Other explanations relate the weaker response of the ECB to inflation dynamics compared to those of the Federal Reserve. Tensions within the ECB Governing Council, a change in the post-GFC inflation target and objectives, quantitative easing, and the zero lower bound could also explain this lower inflation coefficient compared with the Federal Reserve.

Including several SVSs could, at least during crises, more accurately explain the changes in US and EA inflation as well as in US and EA interest rates at different maturities. The possibility of switching in different elements of the economy, such as technology and monetary policy (and not only technology), is essential during crises. It is natural to capture such stylized facts by including several SVSs. Each SVS could capture specific switching volatility that can change the regime of the overall economy for a specific sector. Fig. 6 clearly shows that such modeling is more appropriate for capturing changes in inflation and interest rates than a model with only one SVS for technological progress.

In addition, such shocks are important for capturing changes in an open economy; for instance, Fig. 10 shows that after a US monetary policy shock, EA inflation is assumed to decrease just after the shock in the model without SVSs, whereas this is not the case in reality. Then, models with one or several SVSs could capture reality more accurately, especially during crisis periods when macroeco-

nomic and financial variables are not well explained. Section 4.2 discusses the transmission channels.

Lastly, our models can shed light on nonlinear IRFs, highlighting the significant nonlinear behaviors of market-related variables such as exchange and interest rates. Such dynamics are absent from most policymakers' models for such reasons as technical complexity, material limitations, and time and computational costs. However, our policy recommendation resulting from the results of this study is that nonlinear models should be used when addressing open-economy and market-related variables, which can be subject to highly nonlinear dynamics compared with more standard closed-economy variables.

6 Conclusion

In this study, a two-country open-economy MSDSGE model was developed to understand several stylized events that occurred during the GFC, such as how the regime-specific SVS impacts between the EA and the United States were transmitted to real and financial variables.

Using a second-order approximation and Markov SVS, we showed that SVSs are the main driving force of the shock transmissions during crises. We showed that SVSs affect the US and EA economies and involve i) money transfers between economies and ii) interest rate maturity trade-offs that could produce structural changes in the economy. Hence, SVSs affect US and EA consumption in opposite ways.

Further, price markup and money shocks behave differently to in standard linear models. Owing to direct effects (Andrés et al., 2009), the roles of both domestic and foreign real money holdings are significant in the long run as well as the short run, especially for bond variables and rate-related variables.

Furthermore, the difference between the average response of SVSs and response on specific dates illustrates that SVSs are relevant during crises but less so in calm times. Unlike EU monetary policy, which is less impacted by SVSs, US monetary policy is significantly influenced by such shocks.

The main policy implication relates to the way monetary authorities model the economy, especially in an open-economy world with interlinked financial markets. Our models showed that it is important for policymakers to consider nonlinear models and SVSs during crisis periods (or when uncertainty about a current regime increases). If policymakers continue to use standard linear models and ignore SVSs, they might also overlook some nonlinear dynamics as well as the underlying interactions between financial markets and the economy. SVSs could thus be a promising feature included in the next generation of macroeconomic models.

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Appendix

A Markov switching quadratic Kalman filter

This appendix presents the fast-deterministic filter used for the estimation of our nonlinear MSDSGE model. The collapsing rule of the sigma-point Kalman filters developed by Binning and Maih (2015) is unusual. This filter family uses variance equal to the weighted average of variance conditional on the regime. Such a formula holds for *raw* moments but not for *central* moments.[12] Our MSQKF fixes this property by correcting the formulas for variances.[13] The MSQKF is a Gaussian-assumed filter that uses collapsing before forecasting as in Binning and Maih (2015).

Particle filter approaches have the advantage of an unbiased likelihood estimation. However, these approaches produce a stochastic estimation of likelihood, which is a substantial disadvantage. They do not allow standard optimization algorithms to be used. Moreover, fixed random draws are required for optimization algorithms with particle filters. However, this mitigates the main advantage of particle filters. Markov chain Monte Carlo inefficiency increases significantly: the required number of draws should be 10 (from 5 to 400 depending on the number of particles) times higher for the same accuracy of Markov chain Monte Carlo methods (Pitt et al., 2012). An additional disadvantage of particle filters is their computational costs. They require a large number of particles to be comparable

[12] Let us consider two regimes, the probabilities of which, $p(r_t|r_t+1)$, are $p(1|1) = p(2|2) = 0.95\,(0.6)$ and $p(1|2) = p(2|1) = 0.05\,(0.4)$. The mean conditions on these regimes, $x(r_t)$, are $x(1) = 1$ and $x(2) = -1$, and the variance condition on each regime is 1. Hence, the variance condition on the future regime, $V(r_t+1)$, would be $V(1) = V(2) = 1.19\,(1.96)$, while the formula from Binning and Maih (2015) gives 1 in both cases. This demonstrates that their formula generates substantial errors in the case of regime uncertainty.

[13] The description, properties, and comparisons of the MSQKF are detailed in Ivashchenko (2014, 2016).

with deterministic filters and are about 100 times slower than deterministic nonlinear filters (Andreasen, 2013; Ivashchenko, 2014; Kollmann, 2015). For all these reasons, we do not use particle filters.

The purpose of a filter in DSGE models is to compute the model variable vector, X_t, density conditional on the vectors of the observed variables $Y_1, ..., Y_t$ and density and likelihood of the observed variables $Y_1, ..., Y_t$. Computing the density means computing the parameters of the density approximation. In certain cases, this approximation is equal to the density (e.g., the normal distribution).

Most filters loop the following steps:

1. Computation of the initial density of X_t;
2. Computation of the density of Y_t as a function of the density of X_t (see Appendix A.1);
3. Computation of the likelihood of Y_t (see Appendix A.2);
4. Computation of the conditional density of $X_t|Y_t$ (see Appendix A.3);
5. Computation of the density of X_{t+1} as a function of the density of $X_t|Y_t$ (see Appendix A.4); and
6. Return step 2.

Our MSQKF assumes a Gaussian density approximation in Step 5, unlike the sigma-point one. The sigma-point one is easier to implement for any type of state-space model. The Gaussian one produces a better quality of filtration when the densities are close to the Gaussian ones (Ivashchenko, 2014).

The suggested model of the data-generating process is determined by Eq. 20 to Eq. 22 and a discrete MS process for the regime variable, r_t, where $X_{state,t}$ is the vector of the state variables (a subset of the model variable vector X_t), and ε_t and u_t are the vectors of independent shocks (model innovations and measurement errors) that have a zero-mean normal distribution. δ is a constant equal to one and related to the perturbation with respect to uncertainty. The second-order approximation of the MSDSGE model is computed with the RISE toolbox (Maih, 2015):

$$Y_t = HX_t + u_t, \tag{20}$$

$$Z_t = \begin{bmatrix} X_{state,t} & \delta & \varepsilon_t \end{bmatrix}, \tag{21}$$

$$X_{t+1} = A_{0,r_{t+1}} + A_{1,r_{t+1}} Z_t + A_{2,r_{t+1}} \left(Z_t \otimes Z_t \right), \tag{22}$$

where $\otimes$ is the Kronecker product.

The difference from the usual DSGE model second-order approximation is the existence of regime dependence. Each filtering step is described below. The nonlinear filters (including the suggested ones) use some approximations. Computations within the filters that use approximations are highlighted.

A.1 Density of Y_t as a function of the density of X_t

The initial information for this step is that the density of X_t is a normal mixture. The linear equation for the observed variables, Eq. 20, presents the density of Y_t as a normal mixture with the same probabilities of regimes and the following expectations and variances (conditional on the regime):

$$E_s\left[Y_t\right] = E_s\left[HX_t + u_t\right] = HE_s\left[X_t\right], \tag{23}$$

$$V_s\left[Y_t\right] = V_s\left[HX_t + u_t\right] = HV_s\left[X_t\right]H.' + V_s\left[u_t\right], \tag{24}$$

where $E_s\left[.\right]$ and $V_s\left[.\right]$ denote the expectation and variance operators conditional on regime s.

A.2 Likelihood of Y_t

The initial information for this step is that the density of Y_t is a normal mixture. This means that the likelihood can be determined as

$$\begin{aligned} L\left[Y_t\right] &= \sum_{s=1}^{N_S} p(r_t = s) L\left[Y_t | r_t = s\right] \\ &= \sum_{s=1}^{N_S} p(r_t = s) \frac{e^{-\frac{1}{2}\left(Y_t - E_s[Y_t]\right)' \left(V_s[Y_t]\right)^{-1}\left(Y_t - E_s[Y_t]\right)}}{\left(2\pi\right)^{\frac{N_Y}{2}} \left|V_s\left[Y_t\right]\right|^{\frac{1}{2}}}, \end{aligned} \tag{25}$$

where $L\left[.\right]$ is the likelihood, N_S the number of regimes, and N_Y the number of observed variables.

A.3 Conditional density of $X_t|Y_t$

The initial information for this step is the vector of observation Y_t and that the density of X_t is a normal mixture. The linear Eq. 20 allows a computation conditional on the regime and observation density in the same way as the Kalman filter in Eq. 26 to Eq. 28:

$$K_s' = \left(V_s\left[Y_t\right]\right)^{-1} HV_s\left[X_t\right], \tag{26}$$

$$E_s\left[X_t | Y_t\right] = E_s\left[X_t\right] + K_s\left(Y_t - E_s\left[Y_t\right]\right), \tag{27}$$

$$V_s\left[X_t|Y_t\right] = \left(I_{N_X} - K_s H\right) V_s\left[X_t\right]\left(I_{N_X} - K_s H\right)', \tag{28}$$

$$p\left(r_t = s|Y_t\right) = \frac{p\left(r_t = s; Y_t\right)}{p\left(Y_t\right)} = \frac{p\left(Y_t|r_t = s\right) p\left(r_t = s\right)}{p\left(Y_t\right)}. \tag{29}$$

Eq. 29 shows the probability of regime s conditional on the observed variables. $p\left(Y_t\right)$ is the likelihood (computed in Appendix A.2), and $p\left(Y_t|r_t = s\right)$ has a normal density.

A.4 Density of X_{t+1} as a function of the density of $X_t|Y_t$

The initial information for this step is the density for the vector of the model variables X_t (normal mixture).

The first step is the computation of the expectation ($E_{s,1}$) and variance ($V_{s,1}$) of vector X_t conditional on the future state, such as

$$E_{s,1} = E\left(X_t|r_{t+1} = s\right) = \sum_{k=1}^{N_s} \frac{p\left(r_t = k\right) p\left(r_{t+1} = s|r_t = k\right)}{p\left(r_{t+1} = s\right)} E_k\left(X_t\right), \tag{30}$$

$$\begin{aligned} V_{s,1} &= -E_{s,1}\left(E_{s,1}\right)' \\ &+ \sum_{k=1}^{N_s} \frac{p\left(r_t = k\right) p\left(r_{t+1} = s|r_t = k\right)}{p\left(r_{t+1} = s\right)} \left(E_k\left(X_t\right) E_k\left(X_t\right)' + V_k\left(X_t\right)\right). \end{aligned} \tag{31}$$

The next step is the approximation (collapsing rule): the density of vector X_t is a normal mixture with regime probabilities $p\left(r_{t+1} = s\right)$ and Gaussian densities with moments $E_{s,1}$ and $V_{s,1}$.

The conditional density of X_t provides the density of Z_t. This allows us to compute the conditional moments of the future vector of the variables X_{t+1} ($X_{t+1,r_{t+1}}$ is the future vector of the model variables conditional on future regime r_{t+1}):

$$Z_{0,t,r_{t+1}} = Z_{t,r_{t+1}} - E_{r_{t+1}}\left[Z_{t,r_{t+1}}\right], \tag{32}$$

$$\begin{aligned} X_{t+1,r_{t+1}} &= A_{0,r_{t+1}} + A_{1,r_{t+1}} Z_{t,r_{t+1}} + A_{2,r_{t+1}}\left(Z_{t,r_{t+1}} \otimes Z_{t,r_{t+1}}\right) \\ &= B_{0,r_{t+1}} + B_{1,r_{t+1}} Z_{0,t,r_{t+1}} + B_{2,r_{t+1}}\left(Z_{0,t,r_{t+1}} \otimes Z_{0,t,r_{t+1}}\right) \end{aligned} \tag{33}$$

$$E\left[X_{t+1,r_{t+1}}\right] = B_{0,r_{t+1}} + B_{2,r_{t+1}} vec\left\{V\left[Z_{t,r_{t+1}}\right]\right\} = B_{0,r_{t+1}} + B_{2,r_{t+1}} vec\left\{V_{r_{t+1}}\right\}, \tag{34}$$

$$\begin{aligned} vec\left\{V\left[X_{t+1,r_{t+1}}\right]\right\} &= \left(B_{1,r_{t+1}} \otimes B_{1,r_{t+1}}\right) vec\left\{V_{r_{t+1}}\right\} \\ &+ \left(B_{2,r_{t+1}} \otimes B_{2,r_{t+1}}\right) \left(\begin{array}{l} vec\left\{V_{r_{t+1}}\right\} \otimes vec\left\{V_{r_{t+1}}\right\} + \\ + vec\left(vec\left\{V_{r_{t+1}}\right\} \otimes V_{r_{t+1}}\right) \end{array} \right), \end{aligned} \tag{35}$$

where $vec\{.\}$ is the vectorization operator.

Eq. 32 to Eq. 35 are similar to the equations developed in Ivashchenko (2014). The difference is that these formulas become formulas for moments, conditional on the regime.

The last action of this step is an approximation. The density of X_{t+1} is a normal mixture with moments according to Eq. 34 to Eq. 35.

B Summary of the variables

Table 1 summarizes the variables used in our model, showing the equations in which the variable is used.

Variable	Description	Equations
$B_{i,j,k,t}$	Bonds bought by households i in currency j with maturity k	3, 4, 13, 14
$B_{i,g,t}$	Bonds bought by the central bank or government in country i	11, 13
$C_{i,t}$	Consumption of households of country i	2, 3, 4, 12
$D_{i,t}$	Dividends of firms from country i	6, 7
e_t	Exchange rate (number of units of the domestic currency per unit of the foreign currency)	3, 4, 8, 9, 10
$L_{i,t}$	Labor in country i	2, 4, 5, 7
$M_{i,t}$	Money stock in country i	2, 4, 11
$P_{i,t}$	Aggregate price level in country i	2, 3, 4, 6, 7, 8,9,10
$P_{i,t}(j)$	Price of goods of firms j in country i	6, 7, 8, 9
$R_{i,k,t}$	Interest rate in currency i with maturity k	4, 6, 10, 11
$W_{i,t}$	Wage in country i	4, 7
$Y_{i,t}$	Demand in country i	6, 8, 10, 12
$Y_{F,i,t}(j)$	Production of firms j in country i	5, 7, 8
$\varepsilon^j_{i,t}$	Exogenous process of type j in country i	1, 2, 8, 9, 10, 15
Z_t	Exogenous technology process	2, 5, 6, 15

Table 1: Summary of the variables used in the model's equations

C Estimation results

Table 2 presents the median absolute error (MAE) and log predictive score (LPS) for each observed variable to assess the forecast quality of our models and illustrate

the importance of switching volatility. We compute the LPS based on Gaussian density, which is suggested by the MSDSGE model.[14]

	MAE			**LPS**		
	No SVS	1SVS	3SVS	No SVS	1SVS	3SVS
EA GDP deflator	0.408%	0.366%	0.384%	3.66	3.70	3.67
US GDP deflator	0.156%	0.147%	0.150%	4.58	4.60	4.59
EA 3m rate	0.040%	0.040%	0.041%	5.68	5.68	5.88
US 3m rate	0.086%	0.085%	0.083%	5.35	5.33	5.36
EA demand to GDP	0.611%	0.597%	0.596%	3.43	3.40	3.40
US demand to GDP	0.580%	0.577%	0.587%	3.61	3.63	3.62
EA GDP growth	0.775%	0.734%	0.741%	3.30	3.32	3.31
US GDP growth	0.424%	0.448%	0.416%	3.45	3.46	3.48
EA 10y rate	0.065%	0.067%	0.068%	5.64	5.64	5.64
US 10y rate	0.062%	0.059%	0.054%	5.26	5.29	5.29

Table 2: Forecasting performance for the one-step ahead forecasts.

We also compute these statistics for the 3SVS model in the cases that the volatility state is always in regime 1 and always in regime 2 to illustrate the importance of MS.[15] We show that the 3SVS model when always in regime 1 is much worse in terms of forecasting than that always in regime 2. At the same time, the 3SVS model produces the best density forecasts. The difference in the sum of individual LPSs is relatively small because it does not take into account the correlation between forecasts, which increases the advantage of the 3SVS model.

Tables 3 to 5 present the estimation results for each model. Our results are generally in line with those in the DSGE literature. The persistence of monetary policy shocks is lower than that of other shocks, as explained by Smets and Wouters (2007). The coefficient of relative risk aversion is close to unity and lower than that found in the literature (Benchimol, 2014).

We do not compare the models using their respective log-likelihood ratios for several reasons. First, the difference between the log-likelihood values of the two models does not mean that we must disregard the model with the lowest log-likelihood even if the advantage is statistically significant. For instance, the latter model could still be used to perform forecasting in changing environments (Benchimol and Fourçans, 2017, 2019). Second, whatever the log-likelihood, the model is designed to capture only specific characteristics of the data. It is an open question as to whether log-likelihood is an adequate measure to evaluate how well the model

[14] When based on Gaussian mixture density, the LPS should equal the log-likelihood divided by the number of periods (for the multivariate measure).

[15] See Fig. 1 for the estimated probabilities.

accounts for particular aspects of the data.

Nevertheless, we report the log-likelihood values and corresponding likelihood ratio tests hereafter. The log-likelihood values of the 0SVS, 1SVS, and 3SVS models are 3581.70, 3593.35, and 3611.67, respectively. This means that the p-value of the likelihood ratio test of 1SVS vs. 0SVS is 3.51e-05, 3SVS vs. 1SVS is 1.1e-08, and 3SVS vs. 0SVS is 1.25e-11. Consequently, a more flexible model explains significantly more of the data. Our estimation of the covariance matrix allows us to construct a Laplace approximation of the marginal likelihood (maximum likelihood estimation is equivalent to a Bayesian one with flat priors).

The results are sensitive to the approximation methodology. We use the RISE function "solve_accelerate." If we try to compute the approximation without this function—and compute the likelihood—the resulting values would be 3413.21 (0SVS), 3530.66 (1SVS), and 2515.20 (3SVS). This is probably due to the iterative nature of the MSDSGE solution approximation that converges to a slightly different solution. The sharp likelihood of the nonlinear approximation transforms this small difference into a significant difference in the likelihood. Thus, even *small* details of the solution algorithm can be crucial in a nonlinear world.

	Priors		Posteriors			Priors		Posteriors	
	LB	*UB*	*Mean*	*Std.*		*LB*	*UB*	*Mean*	*Std.*
$\overline{\eta}_{d,u}$	-0.01	0.00	-0.006	0.000	A_f	-20.0	20.0	0.000	0.002
$\overline{\eta}_{d,m}$	-20.0	20.0	-6.493	0.263	$\varphi_{d,d,sr}$	0.00	1000	0.195	0.004
$\overline{\eta}_{d,r}$	0.00	0.01	0.000	0.000	$\varphi_{d,f,sr}$	0.00	1000	0.083	0.000
$\overline{\eta}_{d,p}$	1.00	20.0	3.281	0.020	$\varphi_{d,d,lr}$	0.00	1000	16.26	0.413
$\overline{\eta}_{f,u}$	-0.01	0.00	-0.008	0.000	$\varphi_{d,f,lr}$	0.00	1000	0.236	0.003
$\overline{\eta}_{f,m}$	-20.0	20.0	-7.558	0.001	$\varphi_{f,d,sr}$	0.00	1000	0.001	0.000
$\overline{\eta}_{f,r}$	0.00	0.01	0.001	0.000	$\varphi_{f,f,sr}$	0.00	1000	0.000	0.000
$\overline{\eta}_{f,p}$	1.00	20.0	11.28	0.008	$\varphi_{f,d,lr}$	0.00	1000	0.012	0.001
$\overline{\eta}_{y}$	0.00	0.01	0.001	0.000	$\varphi_{f,f,lr}$	0.00	1000	0.000	0.000
$\eta_{d,u}$	-1.00	1.00	0.978	0.004	s_d	0.00	1.00	0.655	0.006
$\eta_{d,l}$	-1.00	1.00	0.975	0.001	s_f	0.00	1.00	0.063	0.001
$\eta_{d,m}$	-1.00	1.00	0.891	0.007	$\varphi_{d,P}$	0.00	1000	591.4	21.637
$\eta_{d,r}$	-1.00	1.00	0.180	0.021	$\varphi_{f,P}$	0.00	1000	227.8	4.087
$\eta_{d,p}$	-1.00	1.00	0.948	0.001	v_d	0.00	1.00	0.473	0.048
$\eta_{f,u}$	-1.00	1.00	0.462	0.030	v_f	0.00	1.00	0.787	0.008
$\eta_{f,l}$	-1.00	1.00	0.914	0.002	$h_{d,c}$	0.00	0.90	0.669	0.000
$\eta_{f,m}$	-1.00	1.00	0.950	0.003	$h_{f,c}$	0.00	0.90	0.743	0.000
$\eta_{f,r}$	-1.00	1.00	0.756	0.009	$\rho_{d,r}$	0.00	0.99	0.974	0.000
$\eta_{f,p}$	-1.00	1.00	0.953	0.002	$\rho_{d,p}$	1.00	5.00	1.004	0.025
η_{y}	-1.00	1.00	0.960	0.001	$\rho_{d,y}$	-5.00	5.00	0.065	0.005
$\sigma\left(\xi_{d,u}\right)$	0.00	10.0	0.001	0.000	$\rho_{d,e}$	-20.0	20.0	0.005	0.000
$\sigma\left(\xi_{d,l}\right)$	0.00	10.0	0.152	0.007	$\rho_{f,r}$	0.00	0.99	0.796	0.000
$\sigma\left(\xi_{d,m}\right)$	0.00	10.0	0.124	0.004	$\rho_{f,p}$	1.00	5.00	2.906	0.001
$\sigma\left(\xi_{d,r}\right)$	0.00	10.0	0.001	0.000	$\rho_{f,y}$	-5.00	5.00	-0.097	0.003
$\sigma\left(\xi_{d,p}\right)$	0.00	10.0	1.342	0.030	$\rho_{f,e}$	-20.0	20.0	-0.029	0.000
$\sigma\left(\xi_{f,u}\right)$	0.00	10.0	0.007	0.001	$\overline{b}_{d,d,sr}$	-20.0	20.0	0.236	0.006
$\sigma\left(\xi_{f,l}\right)$	0.00	10.0	0.048	0.001	$\overline{b}_{d,f,sr}$	-20.0	20.0	1.859	0.004
$\sigma\left(\xi_{f,m}\right)$	0.00	10.0	0.106	0.002	$\overline{b}_{d,d,lr}$	-20.0	20.0	-0.006	0.000
$\sigma\left(\xi_{f,r}\right)$	0.00	10.0	0.002	0.000	$\overline{b}_{d,f,lr}$	-20.0	20.0	-0.319	0.006
$\sigma\left(\xi_{f,p}\right)$	0.00	10.0	0.635	0.021	$\overline{c}_d$	-20.0	20.0	-0.421	0.000
$\sigma\left(\xi_{y}\right)$	0.00	10.0	0.001	0.000	$\overline{c}_f$	-20.0	20.0	-1.593	0.000
$1/\sigma_{d,c}$	-20.0	20.0	0.727	0.001	$\overline{e}$	-20.0	20.0	1.531	0.001
$1/\sigma_{d,l}$	-20.0	20.0	9.821	0.645	$\overline{p}_d$	0.00	0.01	0.000	0.000
$1/\sigma_{d,m}$	-20.0	20.0	2.723	0.027	$\overline{p}_d(j)$	-20.0	20.0	0.003	0.000
A_d	-20.0	20.0	-0.070	0.143	$\overline{p}_f$	0.00	0.01	0.003	0.000
$1/\sigma_{f,c}$	-20.0	20.0	-0.688	0.000	$\overline{p}_f(j)$	-20.0	20.0	0.000	0.000
$1/\sigma_{f,l}$	-20.0	20.0	-1.758	0.044	$\overline{r}_{d,lr}$	0.00	0.03	0.027	0.000
$1/\sigma_{f,m}$	-20.0	20.0	0.794	0.000					

Table 3: Estimation results for the model without an SVS. *LB* and *UB* stand for lower bound and upper bound, respectively.

	Priors		Posteriors			Priors		Posteriors	
	LB	*UB*	*Mean*	*Std.*		*LB*	*UB*	*Mean*	*Std.*
$p_{reg_t=2\|reg_{t-1}=1}$	0.04	0.35	0.056	0.064	s_d	0.00	1.00	0.599	0.007
$p_{reg_t=1\|reg_{t-1}=2}$	0.04	0.35	0.045	0.028	s_f	0.00	1.00	0.062	0.001
$\overline{\eta}_{d,u}$	-0.01	0.00	-0.006	0.000	$\varphi_{d,P}$	0.00	1000	953.4	15.45
$\overline{\eta}_{d,m}$	-20.0	20.0	-6.613	0.015	$\varphi_{f,P}$	0.00	1000	241.1	4.540
$\overline{\eta}_{d,r}$	0.00	0.01	0.000	0.000	v_d	0.00	1.00	0.543	0.043
$\overline{\eta}_{d,p}$	1.00	20.0	3.255	0.039	v_f	0.00	1.00	0.779	0.010
$\overline{\eta}_{f,u}$	-0.01	0.00	-0.008	0.000	$h_{d,c}$	0.00	0.90	0.670	0.001
$\overline{\eta}_{f,m}$	-20.0	20.0	-7.556	0.000	$h_{f,c}$	0.00	0.90	0.741	0.000
$\overline{\eta}_{f,r}$	0.00	0.01	0.001	0.000	$\rho_{d,r}$	0.00	0.99	0.974	0.000
$\overline{\eta}_{f,p}$	1.00	20.0	11.61	0.021	$\rho_{d,p}$	1.00	5.00	1.002	0.012
$\overline{\eta}_{y}$	0.00	0.01	0.001	0.000	$\rho_{d,y}$	-5.00	5.00	0.077	0.005
$\eta_{d,u}$	-1.00	1.00	0.979	0.001	$\rho_{d,e}$	-20.0	20.0	0.003	0.000
$\eta_{d,l}$	-1.00	1.00	0.971	0.001	$\rho_{f,r}$	0.00	0.99	0.796	0.000
$\eta_{d,m}$	-1.00	1.00	0.897	0.005	$\rho_{f,p}$	1.00	5.00	2.903	0.000
$\eta_{d,r}$	-1.00	1.00	0.197	0.029	$\rho_{f,y}$	-5.00	5.00	-0.087	0.003
$\eta_{d,p}$	-1.00	1.00	0.959	0.000	$\rho_{f,e}$	-20.0	20.0	-0.030	0.000
$\eta_{f,u}$	-1.00	1.00	0.443	0.021	$\overline{b}_{d,d,sr}$	-20.0	20.0	0.249	0.004
$\eta_{f,l}$	-1.00	1.00	0.912	0.005	$\overline{b}_{d,f,sr}$	-20.0	20.0	1.877	0.010
$\eta_{f,m}$	-1.00	1.00	0.952	0.006	$\overline{b}_{d,d,lr}$	-20.0	20.0	-0.007	0.000
$\eta_{f,r}$	-1.00	1.00	0.736	0.006	$\overline{b}_{d,f,lr}$	-20.0	20.0	-0.324	0.003
$\eta_{f,p}$	-1.00	1.00	0.952	0.002	$\overline{c}_d$	-20.0	20.0	-0.421	0.000
η_{y}	-1.00	1.00	0.959	0.002	$\overline{c}_f$	-20.0	20.0	-1.602	0.001
$1/\sigma_{d,c}$	-20.0	20.0	0.733	0.001	$\overline{e}$	-20.0	20.0	1.536	0.000
$1/\sigma_{d,l}$	-20.0	20.0	13.11	0.665	$\overline{p}_d$	0.00	0.01	0.000	0.000
$1/\sigma_{d,m}$	-20.0	20.0	2.856	0.039	$\overline{p}_d(j)$	-20.0	20.0	0.003	0.000
A_d	-20.0	20.0	0.571	0.157	$\overline{p}_f$	0.00	0.01	0.003	0.000
$1/\sigma_{f,c}$	-20.0	20.0	-0.688	0.000	$\overline{p}_f(j)$	-20.0	20.0	0.000	0.000
$1/\sigma_{f,l}$	-20.0	20.0	-1.774	0.048	$\overline{r}_{d,lr}$	0.00	0.03	0.027	0.000
$1/\sigma_{f,m}$	-20.0	20.0	0.793	0.000	$\sigma\left(\xi_{d,u}\right)$	0.00	10.0	0.000	0.000
A_f	-20.0	20.0	-1.872	0.574	$\sigma\left(\xi_{d,l}\right)$	0.00	10.0	0.204	0.008
$\varphi_{d,d,sr}$	0.00	1000	0.192	0.018	$\sigma\left(\xi_{d,m}\right)$	0.00	10.0	0.112	0.003
$\varphi_{d,f,sr}$	0.00	1000	0.082	0.000	$\sigma\left(\xi_{d,r}\right)$	0.00	10.0	0.001	0.000
$\varphi_{d,d,lr}$	0.00	1000	17.51	0.438	$\sigma\left(\xi_{d,p}\right)$	0.00	10.0	1.184	0.017
$\varphi_{d,f,lr}$	0.00	1000	0.244	0.001	$\sigma\left(\xi_{f,u}\right)$	0.00	10.0	0.007	0.000
$\varphi_{f,d,sr}$	0.00	1000	0.001	0.000	$\sigma\left(\xi_{f,l}\right)$	0.00	10.0	0.051	0.001
$\varphi_{f,f,sr}$	0.00	1000	0.000	0.000	$\sigma\left(\xi_{f,m}\right)$	0.00	10.0	0.107	0.002
$\varphi_{f,d,lr}$	0.00	1000	0.008	0.001	$\sigma\left(\xi_{f,r}\right)$	0.00	10.0	0.002	0.000
$\varphi_{f,f,lr}$	0.00	1000	0.000	0.000	$\sigma\left(\xi_{f,p}\right)$	0.00	10.0	0.636	0.010
s_d	0.00	1.00	0.599	0.007	$\sigma\left(\xi_{y\|reg.=1}\right)$	0.00	10.0	0.001	0.000
s_f	0.00	1.00	0.062	0.001	$\sigma\left(\xi_{y\|reg.=2}\right)$	0.00	10.0	0.001	0.000

Table 4: Estimation results for the 1SVS model. *LB* and *UB* stand for lower bound and upper bound, respectively, $p_{reg_t=a|reg_{t-1}=b}$ for the probability of switching to regime a in period t if in regime b in period $t-1$, and $\sigma(\xi_{y|reg.=a})$ for the standard deviation of the corresponding shock in regime a.

	Priors		Posteriors			Priors		Posteriors	
	LB	*UB*	*Mean*	*Std.*		*LB*	*UB*	*Mean*	*Std.*
$p_{reg_t=2\|reg_{t-1}=1}$	0.04	0.35	0.042	0.001	$\varphi_{d,P}$	0.00	1000	796.4	33.12
$p_{reg_t=1\|reg_{t-1}=2}$	0.04	0.35	0.131	0.017	$\varphi_{f,P}$	0.00	1000	218.5	8.054
$\overline{\eta}_{d,u}$	-0.01	0.00	-0.006	0.000	v_d	0.00	1.00	0.615	0.065
$\overline{\eta}_{d,m}$	-20.0	20.0	-6.807	0.054	v_f	0.00	1.00	0.763	0.026
$\overline{\eta}_{d,r}$	0.00	0.01	0.000	0.000	$h_{d,c}$	0.00	0.90	0.666	0.000
$\overline{\eta}_{d,p}$	1.00	20.0	3.253	0.027	$h_{f,c}$	0.00	0.90	0.742	0.001
$\overline{\eta}_{f,u}$	-0.01	0.00	-0.008	0.000	$\rho_{d,r}$	0.00	0.99	0.972	0.000
$\overline{\eta}_{f,m}$	-20.0	20.0	-7.564	0.068	$\rho_{d,\pi}$	1.00	5.00	1.003	0.016
$\overline{\eta}_{f,r}$	0.00	0.01	0.001	0.000	$\rho_{d,y}$	-5.00	5.00	0.067	0.004
$\overline{\eta}_{f,\pi}$	1.00	20.0	11.44	0.100	$\rho_{d,e}$	-20.0	20.0	0.003	0.001
$\overline{\eta}_{y}$	0.00	0.01	0.001	0.000	$\rho_{f,r}$	0.00	0.99	0.796	0.000
$\eta_{d,u}$	-1.00	1.00	0.979	0.001	$\rho_{f,p}$	1.00	5.00	2.901	0.004
$\eta_{d,l}$	-1.00	1.00	0.969	0.002	$\rho_{f,y}$	-5.00	5.00	-0.092	0.008
$\eta_{d,m}$	-1.00	1.00	0.883	0.010	$\rho_{f,e}$	-20.0	20.0	-0.030	0.000
$\eta_{d,r}$	-1.00	1.00	0.184	0.006	$\overline{b}_{d,d,sr}$	-20.0	20.0	0.233	0.006
$\eta_{d,p}$	-1.00	1.00	0.954	0.003	$\overline{b}_{d,f,sr}$	-20.0	20.0	1.867	0.016
$\eta_{f,u}$	-1.00	1.00	0.447	0.027	$\overline{b}_{d,d,lr}$	-20.0	20.0	-0.006	0.000
$\eta_{f,l}$	-1.00	1.00	0.907	0.006	$\overline{b}_{d,f,lr}$	-20.0	20.0	-0.327	0.005
$\eta_{f,m}$	-1.00	1.00	0.954	0.004	$\overline{c}_d$	-20.0	20.0	-0.424	0.003
$\eta_{f,r}$	-1.00	1.00	0.743	0.014	$\overline{c}_f$	-20.0	20.0	-1.597	0.002
$\eta_{f,p}$	-1.00	1.00	0.951	0.005	$\overline{e}$	-20.0	20.0	1.536	0.004
η_{y}	-1.00	1.00	0.958	0.001	$\overline{p}_d$	0.00	0.01	0.000	0.000
$1/\sigma_{d,c}$	-20.0	20.0	0.734	0.001	$\overline{p}_d(j)$	-20.0	20.0	0.003	0.000
$1/\sigma_{d,l}$	-20.0	20.0	10.20	0.790	$\overline{p}_f$	0.00	0.01	0.003	0.000
$1/\sigma_{d,m}$	-20.0	20.0	2.774	0.038	$\overline{p}_f(j)$	-20.0	20.0	0.000	0.000
A_d	-20.0	20.0	-0.015	0.017	$\overline{r}_{d,lr}$	0.00	0.03	0.028	0.000
$1/\sigma_{f,c}$	-20.0	20.0	-0.687	0.002	$\sigma\left(\xi_{d,u}\right)$	0.00	10.0	0.000	0.000
$1/\sigma_{f,l}$	-20.0	20.0	-1.650	0.043	$\sigma\left(\xi_{d,l}\right)$	0.00	10.0	0.176	0.012
$1/\sigma_{f,m}$	-20.0	20.0	0.793	0.000	$\sigma\left(\xi_{d,m}\right)$	0.00	10.0	0.120	0.006
A_f	-20.0	20.0	8.600	4.547	$\sigma\left(\xi_{d,p}\right)$	0.00	10.0	1.235	0.035
$\varphi_{d,d,sr}$	0.00	1000	0.182	0.004	$\sigma\left(\xi_{f,u}\right)$	0.00	10.0	0.007	0.001
$\varphi_{d,f,sr}$	0.00	1000	0.082	0.000	$\sigma\left(\xi_{f,l}\right)$	0.00	10.0	0.047	0.003
$\varphi_{d,d,lr}$	0.00	1000	17.33	1.478	$\sigma\left(\xi_{f,m}\right)$	0.00	10.0	0.103	0.003
$\varphi_{d,f,lr}$	0.00	1000	0.216	0.007	$\sigma\left(\xi_{f,p}\right)$	0.00	10.0	0.651	0.014
$\varphi_{f,d,sr}$	0.00	1000	0.002	0.000	$\sigma\left(\xi_{y\|reg.=1}\right)$	0.00	10.0	0.002	0.000
$\varphi_{f,f,sr}$	0.00	1000	0.000	0.000	$\sigma\left(\xi_{d,r\|reg.=1}\right)$	0.00	10.0	0.000	0.000
$\varphi_{f,d,lr}$	0.00	1000	0.010	0.000	$\sigma\left(\xi_{f,r\|reg.=1}\right)$	0.00	10.0	0.001	0.000
$\varphi_{f,f,lr}$	0.00	1000	0.000	0.000	$\sigma\left(\xi_{y\|reg.=2}\right)$	0.00	10.0	0.001	0.000
s_d	0.00	1.00	0.618	0.010	$\sigma\left(\xi_{d,r\|reg.=2}\right)$	0.00	10.0	0.001	0.000
s_f	0.00	1.00	0.063	0.002	$\sigma\left(\xi_{f,r\|reg.=2}\right)$	0.00	10.0	0.003	0.000

Table 5: Estimation results for the 3SVS model. *LB* and *UB* stand for lower bound and upper bound, respectively, $p_{reg_t=a|reg_{t-1}=b}$ for the probability of switching to regime a in period t if in regime b in period $t-1$, and $\sigma(\xi_{y|reg.=a})$ for the standard deviation of the corresponding shock in regime a.

D Variance decompositions

Tables 6 to 8 present the short- and long-run variance decompositions of the variables with respect to the shocks for each model. The variance decompositions for the nonlinear models require additional comments. The variance decomposition coefficients, $\forall i \in \{d, f\}$ and $\forall j \in \{u, m, l, p, r, y\}$, are computed with respect to the following function:

$$VD_t\left(x, \xi_{i,j}\right) = 1 - \frac{E\left[x_t^2 | \forall s, \xi_{i,j,s} \sim N\left(0,0\right)\right] - \left(E\left[x_t | \forall s, \xi_{i,j,s} \sim N\left(0,0\right)\right]\right)^2}{E\left[x_t^2 | \forall s, \xi_{i,j,s} \sim N\left(0, \sigma\left(\xi_{i,j,s}\right)\right)\right] - \left(E\left[x_t | \forall s, \xi_{i,j,s} \sim N\left(0, \sigma\left(\xi_{i,j,s}\right)\right)\right]\right)^2}, \tag{36}$$

where x_t is the value of the variable of interest (for which the variance decomposition is computed) in period t, $\xi_{I,j}$ is the shock of interest, $\sigma\left(.\right)$ is the standard error operator, $E\left[.\right]$ is the expectation operator, and N is the Normal law. This formula expresses the proportion by which the variance in the variable of interest is lower if the shock of interest is equal to zero in all periods.

The sum of $VD_t\left(x, \xi\right)$ for each shock gives 1 in the case of the linear model. However, this does not hold for nonlinear models. The sum is close to 1 for most of the variables, but there are exceptions.

All the models present variance decompositions in line with the literature. The domestic price markup shock ($\xi_{d,p}$) plays a predominant role in domestic prices ($p_{d,t}$) in both the short run and the long run. However, the domestic preference shock ($\xi_{d,u}$) plays a substantial role in domestic wages ($w_{d,t}$) in the short run and an important role in domestic consumption ($c_{d,t}$) in the long run, which should be greater if we do not consider domestic firms' production, $y_{d,t}\left(j\right)$.

Foreign shocks play a role in the dynamics of several variables, especially the foreign preference shock ($\xi_{f,u}$). This shock drives the dynamics of several foreign as well as domestic variables in the long run, showing that the EA (domestic) is still dependent on the US economy (foreign) and US households' preferences.

Moreover, financial markets are almost entirely dependent on US economy (foreign) shocks and technology progress shocks in all versions of the model in the long term. Foreign shocks play an important role in domestic long-term interest rates, but not for the corresponding bonds in short-term horizons. Thus, the model reproduces the domination by the United States in financial markets.

Interestingly, foreign money demand shocks ($\xi_{f,m}$) play an important role in the dynamics of several domestic variables in the long run (rather than in the short run), such as the exchange rate, domestic long-run interest rates, and most domestic bond quantities, including those bought by central banks. This finding shows the importance of US money demand shocks for the EA's economic dynamics, in line with the closed-economy and linear DSGE literature (Benchimol and Fourçans, 2017).

In addition, foreign worked hours' shocks have a substantial impact on domestic economic dynamics, particularly for bonds' positions and the exchange rate.

The explanatory power of the technological progress shock (ξ_y) is relatively small in the short term. This shock explains 4–6% of domestic output growth and 14–17% of foreign output growth, depending on the model version. This is substantially smaller than the 25.4% for Europe (Lombardo and McAdam, 2012). The long-term explanatory power is 35–43% for domestic output growth and 59–62% for foreign output growth, depending on the model version, which is larger than that found by Lombardo and McAdam (2012) (27.3% for a 20-quarter horizon). A similar picture is related to inflation's explanatory power. The short-term values are 5.7–9.3% for domestic inflation and 13.7–22.5% for foreign inflation, while the long-term values are 19.2–24.7% and 41.8–45.1%, respectively. This differs from the 26.9% and 27.7%, respectively, in the models of the EA (Lombardo and McAdam, 2012). Small open-economy models produce similar long-term values (16â€"21%) for Canada, Spain, and Sweden (Guerrón-Quintana, 2013). However, the long-term explanatory power of the technological progress shock for inflation differs significantly by country: from 13% for Australia to 54% for Belgium (Guerrón-Quintana, 2013).

The variance decomposition of foreign inflation is slightly unusual. The markup shocks are usually crucial for inflation: 64.9% for EA inflation (Guerrón-Quintana, 2013) and more than 80% for US inflation (Smets and Wouters, 2007). However, our model explains 44.9–50.6% of short-term domestic inflation with a domestic markup shock ($\xi_{d,p}$) and 0.4–1.9% of short-term foreign inflation with a foreign markup shock ($\xi_{f,p}$).

There are few differences between the models in terms of the variance decomposition of the variables with respect to structural shocks, except when considering domestic worked hours and domestic money demand shocks. These shocks affect the domestic bonds' positions of households and central banks in different manners, showing that including SVSs in domestic and foreign monetary policies diminishes the role of domestic money demand while increasing the role of worked hours in domestic bonds' positions.

	Short-run variance decompositions											Long-run variance decompositions										
	$\xi_{d,u}$	$\xi_{d,l}$	$\xi_{d,m}$	$\xi_{d,r}$	$\xi_{d,p}$	$\xi_{f,u}$	$\xi_{f,l}$	$\xi_{f,m}$	$\xi_{f,r}$	$\xi_{f,p}$	ξ_{y}	$\xi_{d,u}$	$\xi_{d,l}$	$\xi_{d,m}$	$\xi_{d,r}$	$\xi_{d,p}$	$\xi_{f,u}$	$\xi_{f,l}$	$\xi_{f,m}$	$\xi_{f,r}$	$\xi_{f,p}$	ξ_{y}
$b_{d,d,sr,t}$	0,0	55,2	42,5	0,1	0,0	0,0	0,0	0,0	0,0	0,0	0,2	19,2	2,5	10,9	2,4	15,8	26,4	11,6	10,7	4,1	7,9	17,5
$b_{d,f,sr,t}$	0,2	0,1	0,0	0,2	0,7	44,6	17,1	32,8	3,6	2,3	17,3	3,5	0,0	0,2	1,0	0,8	30,4	17,5	40,8	2,5	12,6	48,9
$b_{d,g,t}$	0,0	55,3	42,4	0,1	0,0	0,2	0,0	0,1	0,1	0,0	0,0	17,6	2,2	10,0	2,1	14,8	27,4	12,5	11,5	4,2	9,4	18,9
$b_{d,d,lr,t}$	1,1	0,0	0,0	5,4	1,4	57,1	17,9	18,2	0,3	1,4	1,6	3,4	0,1	0,2	1,8	4,7	27,0	16,0	39,2	3,1	13,1	48,2
$b_{d,f,lr,t}$	0,1	0,1	0,0	0,2	0,0	70,9	20,0	0,6	4,9	2,1	21,6	4,2	0,1	0,2	1,1	3,2	27,3	15,7	42,5	2,4	15,9	47,1
$b_{f,g,t}$	0,5	0,0	0,0	0,5	0,0	65,5	21,0	1,9	7,8	2,0	29,5	3,7	0,0	0,0	1,1	0,2	24,3	13,1	34,6	2,9	4,9	72,3
$c_{d,t}$	7,8	18,7	0,9	69,4	5,6	0,2	0,1	0,1	0,0	0,9	1,5	26,5	0,4	0,1	2,3	39,5	13,2	7,2	15,0	1,6	13,2	16,1
$c_{f,t}$	0,2	0,0	0,0	0,1	0,5	46,0	0,3	18,0	21,3	2,0	8,7	1,6	0,1	0,0	0,0	1,0	61,4	10,4	22,2	2,7	24,5	20,0
e_{t}	6,0	1,7	0,0	2,0	3,0	41,5	17,9	24,5	4,0	1,3	16,6	2,3	0,1	0,1	1,4	1,2	40,3	19,4	31,6	5,8	12,3	41,4
$m_{d,t}$	3,2	54,4	37,5	1,5	4,2	0,4	0,0	0,1	0,1	0,3	0,0	10,3	5,1	13,9	9,7	12,4	27,2	11,5	9,4	3,6	5,9	18,9
$m_{f,t}$	0,3	0,0	0,0	0,5	0,0	54,5	24,9	0,4	5,7	1,7	19,2	2,6	0,1	0,1	1,5	0,8	31,8	18,3	34,7	4,3	8,5	54,8
$p_{d,t}$	34,8	4,2	0,7	25,8	50,6	1,0	0,2	0,3	0,1	5,0	5,8	12,7	1,4	2,4	10,5	53,5	5,7	1,9	4,4	0,4	10,1	19,2
$p_{f,t}$	0,6	0,0	0,0	0,2	0,0	38,4	1,9	38,3	3,3	1,9	13,7	2,2	0,1	0,1	1,4	1,0	37,6	18,9	33,0	5,2	10,4	43,8
$r_{d,t}$	0,6	0,1	98,6	0,6	0,5	0,0	0,0	0,0	0,0	0,0	0,3	13,7	0,3	16,5	2,5	11,4	29,5	13,0	10,3	4,4	6,8	20,5
$r_{f,t}$	0,6	0,0	0,0	0,5	0,0	64,3	1,6	0,0	8,8	2,1	22,3	2,4	0,1	0,0	1,4	1,0	36,8	17,9	28,1	5,4	9,5	48,4
$r_{d,lr,t}$	0,7	28,3	11,1	3,5	0,7	35,3	7,2	9,5	0,2	0,7	0,4	4,8	0,2	1,4	1,5	2,6	37,6	16,5	32,5	4,6	13,6	39,9
$r_{f,lr,t}$	2,6	0,0	0,0	0,6	1,8	16,5	3,9	12,4	0,3	1,1	63,8	2,2	0,1	0,1	1,3	1,2	38,0	19,3	29,9	5,6	10,8	45,0
$w_{d,t}$	64,4	2,8	0,1	14,2	2,3	1,7	0,8	1,4	0,0	15,9	1,0	5,7	0,3	0,1	4,3	72,2	3,0	2,2	7,2	0,0	17,4	7,0
$w_{f,t}$	0,0	0,0	0,0	0,1	0,1	62,3	0,7	10,2	15,4	3,4	3,8	1,2	0,2	0,0	0,0	0,2	40,1	3,3	23,0	7,3	30,8	19,8
$y_{d,t}(j)$	2,8	6,6	0,1	28,3	4,9	6,0	2,4	5,6	0,0	56,1	4,2	30,5	0,4	0,1	2,6	51,8	2,8	2,4	7,5	0,0	16,6	6,4
$y_{f,t}(j)$	0,5	0,2	0,0	0,0	3,8	29,3	1,6	14,1	16,1	34,7	5,6	1,8	0,3	0,0	0,3	1,3	63,4	5,9	16,0	2,2	30,9	15,3

Table 6: Short- and long-run variance decompositions for the model without an SVS

	Short-run variance decompositions											Long-run variance decompositions										
	$\xi_{d,u}$	$\xi_{d,l}$	$\xi_{d,m}$	$\xi_{d,r}$	$\xi_{d,p}$	$\xi_{f,u}$	$\xi_{f,l}$	$\xi_{f,m}$	$\xi_{f,r}$	$\xi_{f,p}$	ξ_{y}	$\xi_{d,u}$	$\xi_{d,l}$	$\xi_{d,m}$	$\xi_{d,r}$	$\xi_{d,p}$	$\xi_{f,u}$	$\xi_{f,l}$	$\xi_{f,m}$	$\xi_{f,r}$	$\xi_{f,p}$	ξ_{y}
$b_{d,d,sr,t}$	0,0	50,2	49,4	0,3	0,0	0,0	0,0	0,0	0,0	0,0	0,3	17,6	2,2	13,8	1,7	13,1	24,7	11,8	10,7	2,4	11,1	19,2
$b_{d,f,sr,t}$	0,1	0,1	0,1	0,5	0,4	43,5	16,9	30,5	4,2	1,9	16,9	0,7	0,1	0,0	1,3	0,7	28,5	16,5	36,5	3,3	13,6	55,7
$b_{d,g,t}$	0,0	50,1	49,3	0,3	0,0	0,2	0,1	0,1	0,0	0,0	0,4	14,7	1,9	12,2	1,6	11,7	25,9	12,4	11,9	2,8	13,0	22,5
$b_{d,d,lr,t}$	0,5	0,0	0,0	4,4	0,7	59,9	20,1	14,4	0,7	0,7	2,7	3,7	0,3	0,0	2,9	4,3	23,6	17,4	37,6	3,3	12,0	49,8
$b_{d,f,lr,t}$	0,3	0,1	0,0	0,4	0,0	68,6	16,1	0,6	5,5	0,6	24,6	1,4	0,2	0,0	2,1	2,0	25,1	16,1	39,1	3,3	15,8	51,8
$b_{f,g,t}$	0,6	0,0	0,0	1,2	0,0	63,1	16,8	1,8	8,9	0,6	32,0	1,2	0,0	0,0	1,5	0,9	25,7	12,2	29,9	1,7	6,0	77,0
$c_{d,t}$	7,0	17,4	0,6	74,2	3,2	0,3	0,1	0,0	0,0	0,5	1,3	24,3	0,9	0,1	3,8	28,3	16,3	8,5	17,0	0,7	18,1	23,0
$c_{f,t}$	0,2	0,0	0,0	0,2	0,4	49,8	0,5	18,1	22,2	1,6	8,8	0,7	0,1	0,1	2,0	1,4	58,8	7,4	19,4	2,4	25,5	25,3
e_{t}	3,9	1,4	0,0	2,8	1,6	46,3	19,6	22,2	4,9	0,6	19,3	1,8	0,0	0,0	2,8	1,4	39,9	22,9	31,1	5,0	11,2	40,0
$m_{d,t}$	2,8	50,1	43,5	0,9	3,4	0,4	0,1	0,1	0,0	0,2	0,2	10,8	4,3	18,6	11,4	10,3	22,5	11,3	8,1	2,3	7,8	16,4
$m_{f,t}$	0,5	0,0	0,0	0,8	0,1	51,8	22,0	0,6	6,6	0,5	23,8	1,5	0,0	0,0	2,3	1,1	32,6	18,7	31,7	3,4	8,0	54,1
$p_{d,t}$	36,0	3,7	0,0	22,3	44,9	1,8	0,0	1,0	0,0	5,3	9,3	16,5	2,2	1,5	10,6	42,0	6,8	4,1	7,6	0,6	14,5	23,8
$p_{f,t}$	0,5	0,0	0,0	0,8	0,1	39,2	0,8	36,2	4,6	0,4	18,6	1,8	0,0	0,0	2,5	1,1	38,8	20,9	32,7	4,7	10,0	41,8
$r_{d,t}$	0,5	0,0	98,8	0,3	0,5	0,1	0,0	0,0	0,0	0,0	0,2	13,5	0,0	21,8	1,4	10,1	25,9	12,3	8,6	2,7	8,6	19,0
$r_{f,t}$	0,5	0,0	0,0	0,9	0,1	64,6	0,4	0,2	8,8	0,6	26,0	1,3	0,0	0,0	2,3	1,0	38,7	18,7	26,5	3,9	8,3	46,6
$r_{d,lr,t}$	0,3	21,7	11,9	3,3	0,6	44,9	9,3	6,7	0,1	0,4	2,4	4,2	0,3	0,4	2,9	1,9	34,5	19,7	32,9	4,5	13,5	41,5
$r_{f,lr,t}$	1,4	0,0	0,0	1,3	0,8	14,3	3,6	10,8	0,6	0,0	69,0	1,6	0,0	0,0	2,5	1,2	38,8	21,9	29,4	4,7	9,8	42,9
$w_{d,t}$	66,7	1,9	0,1	11,0	0,6	1,7	0,4	2,7	0,0	16,6	1,8	11,5	1,2	0,1	6,1	57,3	5,3	3,4	9,3	0,1	25,7	9,3
$w_{f,t}$	0,2	0,1	0,0	0,2	0,2	64,5	0,2	10,8	16,6	3,0	5,4	1,1	0,2	0,0	1,1	0,3	42,3	4,4	18,4	6,9	32,6	20,6
$y_{d,t}(j)$	3,4	4,7	0,0	29,1	2,3	7,1	3,2	8,9	0,2	60,6	5,9	33,0	0,8	0,2	4,5	40,5	4,2	2,3	8,5	0,1	22,5	8,6
$y_{f,t}(j)$	0,3	0,3	0,0	0,6	1,8	33,3	2,0	15,1	15,3	34,0	6,3	2,2	0,0	0,0	1,1	0,2	64,6	7,3	12,9	1,4	30,6	14,4

Table 7: Short- and long-run variance decompositions for the 1SVS model

	Short-run variance decompositions											Long-run variance decompositions										
	$\xi_{d,u}$	$\xi_{d,l}$	$\xi_{d,m}$	$\xi_{d,r}$	$\xi_{d,p}$	$\xi_{f,u}$	$\xi_{f,l}$	$\xi_{f,m}$	$\xi_{f,r}$	$\xi_{f,p}$	ξ_{y}	$\xi_{d,u}$	$\xi_{d,l}$	$\xi_{d,m}$	$\xi_{d,r}$	$\xi_{d,p}$	$\xi_{f,u}$	$\xi_{f,l}$	$\xi_{f,m}$	$\xi_{f,r}$	$\xi_{f,p}$	ξ_{y}
$b_{d,d,sr,t}$	0,0	76,4	23,2	0,4	0,0	0,1	0,0	0,0	0,0	0,0	0,0	18,4	2,5	13,2	2,9	14,9	25,2	13,0	9,7	3,3	7,8	20,5
$b_{d,f,sr,t}$	0,8	0,0	0,0	0,3	0,8	51,0	18,1	19,3	4,1	2,3	19,7	3,4	0,1	0,2	2,6	0,7	31,3	22,2	39,7	2,7	13,2	51,1
$b_{d,g,t}$	0,0	76,5	23,0	0,4	0,0	0,0	0,0	0,1	0,0	0,0	0,1	16,2	2,2	11,9	2,8	13,7	26,6	14,2	10,6	3,7	9,6	22,4
$b_{d,d,lr,t}$	1,4	0,0	0,0	4,6	1,8	58,8	19,4	10,9	0,6	0,9	1,9	5,4	0,0	0,0	3,6	3,7	24,0	21,9	39,9	1,9	10,7	51,2
$b_{d,f,lr,t}$	0,6	0,0	0,0	0,0	0,0	68,7	16,3	0,4	5,2	2,1	25,7	5,0	0,0	0,1	3,6	1,6	25,8	22,0	41,4	2,1	15,5	50,2
$b_{f,g,t}$	1,2	0,0	0,0	0,2	0,0	63,6	17,7	1,4	8,6	2,0	34,1	0,3	0,1	0,2	1,1	0,6	31,6	13,2	33,5	2,1	2,9	73,4
$c_{d,t}$	8,4	16,0	0,2	73,5	7,3	0,6	0,1	0,0	0,0	0,2	0,9	26,0	0,6	0,5	3,9	35,2	14,6	10,3	14,4	1,1	13,7	18,6
$c_{f,t}$	0,1	0,0	0,0	0,3	0,6	49,2	0,8	9,9	26,6	2,8	10,9	1,3	0,1	0,2	1,5	0,6	56,6	10,3	20,2	2,5	25,0	25,2
e_{t}	6,0	1,5	0,0	2,7	2,9	44,7	22,6	16,7	4,3	1,0	21,0	1,9	0,0	0,0	2,7	0,6	37,9	23,4	33,5	4,1	8,7	43,5
$m_{d,t}$	3,1	72,8	20,7	1,6	3,9	0,0	0,0	0,1	0,0	0,6	0,4	13,5	4,6	15,9	10,8	12,0	21,4	9,9	8,4	2,7	4,9	19,9
$m_{f,t}$	0,8	0,0	0,0	0,3	0,0	54,0	22,1	0,7	5,9	1,6	24,5	1,1	0,0	0,1	2,0	0,4	33,0	20,7	33,9	3,3	5,0	56,2
$p_{d,t}$	40,9	3,1	0,6	22,7	46,3	0,2	0,0	0,2	0,1	5,4	8,7	15,3	1,7	2,0	9,6	48,9	5,0	2,4	4,9	0,5	12,1	24,8
$p_{f,t}$	0,9	0,1	0,0	0,7	0,1	45,1	0,7	27,6	4,8	1,8	22,5	1,6	0,0	0,0	2,1	0,4	36,8	21,3	33,1	3,8	7,7	45,1
$r_{d,t}$	0,9	0,1	96,0	1,0	2,4	0,0	0,0	0,0	0,0	0,0	0,1	15,1	0,0	19,0	2,3	11,7	25,0	12,0	8,6	3,5	5,5	23,1
$r_{f,t}$	0,6	0,0	0,0	0,4	0,0	64,4	0,3	0,4	8,7	1,7	26,5	0,9	0,0	0,1	1,9	0,3	36,1	19,6	27,7	4,6	5,6	49,7
$r_{d,lr,t}$	0,6	31,4	4,6	2,9	1,2	44,6	8,5	6,2	0,5	1,2	1,4	5,2	0,1	1,1	3,3	1,9	32,9	21,7	36,6	3,0	11,2	43,3
$r_{f,lr,t}$	2,7	0,1	0,0	0,9	1,2	15,1	2,3	7,6	0,8	0,7	72,4	1,6	0,0	0,0	2,2	0,5	36,5	22,3	30,1	4,3	7,2	46,1
$w_{d,t}$	73,6	1,6	0,1	9,9	1,9	1,2	0,9	1,1	0,1	17,3	1,5	7,4	0,9	0,3	6,0	63,3	2,7	2,0	6,9	0,7	22,1	9,3
$w_{f,t}$	0,2	0,0	0,0	0,2	0,3	65,6	0,2	7,7	16,9	2,7	6,6	0,3	0,0	0,0	0,6	0,1	41,0	2,4	21,5	10,0	33,0	19,2
$y_{d,t}(j)$	2,9	4,4	0,0	26,4	4,1	6,1	3,9	2,4	0,0	62,7	5,1	27,6	0,4	0,3	4,4	46,2	3,3	1,4	6,0	0,9	19,8	10,0
$y_{f,t}(j)$	0,8	0,2	0,0	0,4	3,5	32,8	2,3	8,7	17,9	37,0	7,4	1,1	0,0	0,1	0,9	0,9	59,1	6,7	14,0	3,0	30,2	19,4

Table 8: Short- and long-run variance decompositions for the 3SVS model