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Competitive Elections, Incumbency Advantage, and Accountability

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Abstract

I present a model of repeated electoral competition between two parties. Parts of the electorate vote retrospectively and consider the amount of rent-seeking by the incumbent party, while the prospective voters follow probabilistic party preferences when casting their votes. I show that it is possible to distinguish the effects of incumbency advantage and electoral punishment on the minimum level of rent-seeking that is consistent with equilibrium. As long as there is electoral punishment for excessive rent-seeking, a larger incumbency advantage increases accountability by decreasing the minimum amount of rent-seeking consistent with equilibrium. The reason is that the larger the incumbency advantage is, the more important is the result of the next election for all future election outcomes. Consequently, the incumbent party is willing to give up more rent-seeking opportunities to improve its electoral prospects.

JEL: D72, Keywords: Incumbency Advantage, Accountability, Competitive Elections, Probabilistic voting, Rent-seeking

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1 Introduction

I present a model of repeated political competition between two parties and show that an increase in the incumbency advantage increases political accountability. The requirement for holding politicians accountable are not closely fought elections but that incumbents are punished at the ballot box for excessive rent-seeking. These results seem inconsistent with the widely held belief that competitive elections are necessary for a well-functioning democracy, a view that goes back at least to Schumpeter (1942).

However, there is no general agreement about what actually constitutes a competitive election. Brunell and Clarke (2012, 125) give a good description of the widespread understanding which equates competitiveness with small margins of victory when they note that "academics, journalists, and other commentators constantly extol the virtues of competitive elections while bemoaning the fact that there are not enough closely fought contests for important offices." Buchler (2007) proposes to define competitive elections as elections with a close to 50% chance of both sides to win. These views do not take into account, however, that a competitive election is not an end in itself but desirable for a specific reason. It provides incentives for the politicians to work in the interest of the electorate.

Consequently, a better measure of competitiveness is provided by what I define as electoral punishment. The *electoral punishment* is the loss in the likelihood of re-election when the incumbent party does not fulfill a performance standard set by the electorate. In the case at hand, this performance standard is a maximal amount of acceptable rent-seeking. I show that the amount of rent-seeking the voters have to accept in equilibrium is decreasing in the electoral punishment.

More surprising is the main finding that for a given amount of electoral punishment, a larger incumbency advantage leads to less rent-seeking. The *incumbency advantage* is the difference between the probability of remaining in office and the probability of losing office *along the equilibrium path of the model*.¹ Therefore, elections with a close to 50% chance of an opposition victory lead, *ceteris paribus*, to less accountability than elections with a sizable incumbency advantage, usually considered to be more competitive. The mechanism behind this result is the following: The larger the incumbency advantage is, the larger is the influence of the next election

¹This standard definition of the incumbency advantage follows, among others, Besley (2006).

on all future election outcomes. Therefore, the incumbent party is willing to give up more rent-seeking opportunities to improve its electoral prospects, and the electorate can impose a more stringent performance standard.

In the second part of the analysis, I show that a considerable amount of electoral punishment and a large incumbency advantage are not only compatible with each other but naturally coincide if there is a sufficient share of retrospective voters. These retrospective voters cast their votes depending on the amount of rent-seeking preceding the election and vote for the incumbent party if and only if it has complied with the performance standard. The second part of the electorate is formed by the prospective voters. They have preferences for one of the two parties to win office and vote accordingly. These preferences are determined by a random process before each election. While the details are not modelled explicitly, the preferences of the prospective voters can be thought of to depend on the policy platforms or the perceived quality of the candidates. Because of the random element in the voting decision of the prospective voters, the setup constitutes a repeated probabilistic voting model. There is additional variation in the size of the incumbency advantage due to an electoral bias of the prospective voters either for or against incumbent parties.

Because the retrospective voters always reward the incumbent party for compliance, sometimes the incumbent party is re-elected although the majority of the prospective voters prefers the opposition party to win. The probability that this happens is increasing in the share of retrospective voters. Therefore, a larger share of retrospective voters does not only increase accountability but is likely to come at a cost in other dimensions of politics, especially if the two groups of voters differ only in their voting strategies and have similar preferences over policies and candidates.² Interestingly, if the reduced influence of prospective voters leads to a slowdown in the introduction of new ideas and especially new political leaders, this does constitute a reduction of competition in the sense of Schumpeter (1942) whose dynamic conception of political competition is closely related to his economic one.

The following three cases demonstrate the relevance of the results for comparative political economics. The first one considers mature democracies, which usually combine a considerable amount of incumbency advantage with a credible opposition that

²Strictly speaking, the model discussed here does not allow for the two groups of voters to have similar preferences over policies and candidates because the utility of the retrospective voters depends only on the amount of rent-seeking. However, this assumption could easily be relaxed without changing any major results.

allows for electoral punishment at the ballot box and corresponds with a considerable share of retrospective voters, possibly in combination with some additional electoral bias in favor of the incumbent party.

I discuss Germany as an example. Since the first postwar elections in 1949, only candidates of the two major parties CDU and SPD have become German chancellor, and just as in the model discussed here two parties repeatedly compete for this office.³ Moreover, there is evidence for a sizable incumbency advantage: In 15 out of 18 elections, the incumbent chancellor remained in office after the election. This is not because the CDU candidates were more often elected: of the five elections with an SPD chancellor as the incumbent candidate, the SPD won four.⁴ While this large incumbency advantage brought stability and accountability, there might have been a certain lack of new policy ideas. This is especially true for the period before the first SPD chancellorship after the election in 1969 and consistent with the implication of the model that a large degree of accountability can come at the cost of the reduced influence of prospective voters. German economic policy provides one example. Keynesian ideas, for better or worse, arrived comparatively late in German economic policymaking and only after the SPD, for the first time, became part of the Government in 1966. Keynesian economics had achieved a strong position in German academia already much earlier (Hagemann 2017). A list with the German elections and their results is provided in Appendix A.⁵

The second case is provided by countries with reasonable fair and competitive elections but low incumbency advantage or even disadvantage and consequently also at best moderate degrees of electoral punishment. This corresponds to a small share of retrospective voters in combination with an electoral bias for the opposition. The Central and Eastern European countries after the restoration of democracy provide an example. Roberts (2008) considers 34 free elections between 1992 and 2006 in 10 different countries in Central and Eastern Europe and finds what he calls "hy-

³CDU is short for Christian Democratic Union of Germany, considered to be center-right. SPD is short for Social Democratic Party of Germany, considered to be center-left. The Federal Chancellor of the Federal Republic of Germany is the head of government of Germany. The role of the German Chancellor is comparable to that of the Prime Minister in most other parliamentary democracies.

⁴Neither seems it to be a personal incumbency advantage: While CDU and SPD replaced their chancellors for another party member between general elections three times, only Kurt Georg Kiesinger lost his chancellorship in an election following such a replacement.

⁵For an up to date introduction to the German political system see Marschall (2018). An alternative in English is provided by Schmidt (2003).

peraccountability". However, this "hyperaccountability" amounts in the end to none or little accountability from a moral hazard perspective. While Roberts finds that better economic performance and especially lower unemployment were rewarded in elections, he also reports that the expectations were so high that only about 30% of parties in government before an election remained in government after the election took place. This evidence suggests that the voters were looking for parties that could handle the economy and especially the labor market successfully but were mostly unable to find them. Because the voters used past performance as indicative for future performance, this form of economic voting should be interpreted as prospective. It had the consequence of making elections an ineffective tool for holding politicians accountable for corruption. Or, as Roberts (2008, 533) puts it: "If incumbents know they will lose, then they may decide to enrich themselves when in power".

The third case combines even larger incumbency advantage with low or nonexistent electoral punishment and thus at best minimal accountability. There are many examples for this combination because it naturally occurs in countries in which elections are routinely rigged in favor of the government and large parts of the press are controlled by it. Russia under President Putin is currently providing the most prominent example.

Related literature Important early contributions which argue for the empirical importance of retrospective voting are Key (1966) and Fiorina (1981). Formal models of electoral accountability were first developed by Barro (1973) and Ferejohn (1986). In this literature, politicians are the agents, and the voters are their principals. The model presented here is in line with the focus on moral hazard problems, specifically rent-seeking and effort provision by the politicians, of these early models. Except for Klingelhöfer (2015), the idea of having two parties that alternate in office has been neglected although this setup seems a natural choice for analyzing, for example, elections in the United States with its two dominating parties. Instead, the focus is on individual politicians who lose office forever when they lose an election.

An influential critique of the focus on punishing past behavior in models of political accountability is provided by Fearon (1999) who points out the difficulty of holding politicians accountable for past actions in an election if the challenger is not perceived as an equally attractive candidate by the electorate. In this view, rational voters will always vote in favor of candidate or party who would maximize their future utility

if elected and thus consider past behavior only in so far as it is informative about the expected future performance of the incumbent. Fearon’s contribution resulted in an increased focus on the selection of politicians instead of the previous focus on moral hazard problems. However, Ashworth, Bueno de Mesquita, and Friedenber (2017) show that while there can be a trade-off between selecting good-types and holding politicians accountable for past actions, up to a point both aims can be achieved simultaneously. Moreover, as Feddersen and Sandroni (2006) point out, the probability that the decision of a single voter is pivotal is negligible in national elections. In the model presented here, the electorate is represented by a continuum of voters, and just as in real-world national elections, no voter ever casts a decisive vote. Although the retrospective voters know that they are never pivotal, they are nonetheless strategic in the sense that they optimize over their performance standard. That retrospective voters can achieve the coordination on a voting rule that is necessary to implement a common rent-seeking threshold is a standard assumption in the literature on political accountability.

The number of models that combine prospective and retrospective voting motives is small.⁶ Klingelhöfer (2015) shows that when accountability and policy determination are analyzed separately and, besides, there is uncertainty over the voters’ party preferences, important interdependencies between the two dimensions will be overlooked. Specifically, because indifferent voters vote for the incumbent party as long as it limits its rent-seeking, the opposition party tries to differentiate itself on the policy dimension to have a chance to win the election. This effect cannot be shown in any purely forward- or backward-looking model of policy determination. Van Weelden (2013) shows how the electorate can use the policy dimension to increase accountability. To achieve this, a representative voter increases the punishment effect of not re-electing an incumbent by replacing her with a politician from the opposite political spectrum and in this way decrease rent-seeking. While voters suffer from more polarized equilibrium policies, they are nonetheless better off because the increase in accountability has a larger effect on their welfare. However, in contrast to the model presented here, neither of these models allows for different groups of voters with different voting motives. Instead, they allow all voters to combine retrospec-

⁶Persson and Tabellini (2000), an influential textbook in theoretical political economics, treats accountability and policy determination in separate chapters without considering any possible interaction.

tive and prospective voting motives. While rationality requires the voters to vote prospectively whenever they are not indifferent between the candidates, competition on the policy dimension ensures that this is regularly the case. A subsequent paper that provides some additional results is Van Weelden (2015).

The empirical existence of incumbency advantage in many advanced economies is well documented, especially in the United States. Eggers (2017) provides a short and up to date review of the relevant literature. Klašnja and Titunik (2017) provides a convincing attempt to explain incumbency disadvantage in Brazilian mayoral elections. Besides empirical evidence, they also provide a model in which the lack of accountability leads to incumbency disadvantage. More specifically, in their model, weak accountability is due to the insufficient control over politicians by the parties that nominate them. Therefore, low accountability reduces the incumbency advantage, the opposite direction of causality compared to the model presented here. Klašnja and Titunik's results are nonetheless complementary to the ones provided here because both directions of causality could be relevant simultaneously.

Outline of the paper In Section 2, the basic model of political accountability is presented and three different performance standards are derived and discussed. In Section 3, an explicit model of the electorate is provided and the conditional re-election probabilities of Section 2 are endogenized. The paper ends with a conclusion.

2 Model

There are two parties, A and B . The incumbent party in period t is denoted by $I_t \in \{A, B\}$, while the opposition party is denoted by $O_t \in \{A, B\}$ with $O_t \neq I_t$. Without loss of generality, party A is the incumbent party in the first period ($I_0 = A$). In each period t , the incumbent party I_t chooses its amount of rent-seeking. The maximally possible amount of rent-seeking is normalized to 1 and thus $r_t \in [0, 1]$ in all t .⁷ The opposition party cannot engage in any rent-seeking until it wins an election and becomes the new incumbent party. An election takes place in every period after

⁷A more general framework would allow work benefits of office and shirking besides the rent-seeking aspect. For simplicity, I focus on the aspect rent-seeking throughout. This does not lead to any loss of generality because the rents can be thought of as the benefits of office net of the costs of effort provided by the incumbent party and its politicians. Providing less effort than is optimal from the perspective of the voters is a form of shirking by the parties.

the amount of rent-seeking r_t is observed. A representative retrospective voter, short *RRV*, decides whether to vote for the incumbent party in period t ($v_t = 1$) or not ($v_t = 0$). If $v_t = 1$, the incumbent party wins the election with probability α_c , if $v_t = 0$, the incumbent party wins with probability $\alpha_d \leq \alpha_c$. The winner of the election becomes the new incumbent party I_{t+1} . After the election has taken place, the new period $t + 1$ begins with the new incumbent party choosing its amount of rent-seeking. Therefore, every period $t \geq 0$ constitutes one stage of an infinitely repeated stage game. All actions are observed by both parties as well as the *RRV*. Party $J = A, B$ maximizes the expected discounted value of its rent-seeking activities:

$$U^J = E_0 \sum_{t=0}^{\infty} \beta^t 1_J(I_t) r_t, \quad (1)$$

where $\beta \in (0, 1)$ is the discount factor. $1_J(I_t)$ is an indicator function that takes the value 1 in each period t in which party J is in office and value 0 in each period in which party J is out of office. The *RRV* minimizes the value of the expected discounted rent-seeking activities of the parties:

$$U^{RRV} = E_0 \sum_{t=0}^{\infty} \beta^t (1 - r_t). \quad (2)$$

Therefore, in every stage of the game, a total payoff 1 is divided between the incumbent party of the period and the electorate and the game is zero-sum.⁸ The reason why the *RRV* only partly determines the election outcome is the existence of prospective voters. Their voting decisions are discussed in Section 3 where an explicit model of the voting stage is added. There it is shown how the re-election probabilities α_d and α_c depend on attributes of the electorate.

2.1 Solving the model

The equilibrium concept applied throughout is subgame perfection. Following the literature going back to Barro (1973) and Ferejohn (1986), the *RRV* sets a performance

⁸The zero-sum property is made to simplify the analysis as well as the notation and should not be interpreted as implying that real-world rent-seeking has no other further undesirable effects besides the redistribution of resources. It would be straightforward to assume socially wasteful rent-seeking without changing any essential features of the model. However, this would come at some cost of readability.

standard for the incumbent party on which her voting decision depends. In the model presented here, this performance standard is the largest amount of rent-seeking $\bar{r}_t$ in period t for which the RRV casts her vote in favor of the incumbent party I_t . First, equilibria in which the threshold $\bar{r}_t$ is constant over time are considered. After deriving the most stringent constant performance standard consistent with compliance of the parties in equilibrium, I show that this performance standard is also consistent with noncompliance of both parties. The plausibility of the different equilibria with and without compliance is discussed using a forward induction argument which shows that noncompliance is more plausible when both, compliance and noncompliance, are consistent with equilibrium. Therefore, an optimizing electorate which is restricted to constant performance standards chooses a less demanding performance standard which leads to compliance. Finally, an alternative type of equilibrium in which the threshold $\bar{r}_t$ depends on the history of the game is discussed. In this type of equilibrium, the electorate can implement the lowest amount of rent-seeking consistent with rent maximizing parties while ruling out noncompliance.

Constant threshold with compliance equilibrium

Definition 1 *Constant threshold with compliance strategy profile (CTCSP):*

$$\bar{r} \in [0, 1],$$

$$s_J : r_t = \bar{r} \text{ in each } t \text{ in which } I_t = J \text{ with } J = A, B,$$

$$s_{RRV} : v_t = \begin{cases} 1 & \text{if } r_t \leq \bar{r} \\ 0 & \text{if } r_t > \bar{r} \end{cases} \text{ in all } t.$$

Here, s_J denotes the strategy of party J and s_{RRV} the strategy of the RRV . Because incumbent parties are always complying with $\bar{r}$, they are re-elected with probability α_c . The strategy of the RRV is a best response because, given the strategy followed by the parties, she is always indifferent which party wins the election. I show that for a large enough threshold $\bar{r}$, the strategy of the parties is also optimal, and thus, the proposed strategy profile constitutes a subgame perfect *Constant threshold with compliance equilibrium* or *CTCE*. If the *CTCSP* is played, the probability p_s^J of party J being in office in period s given the probability p_{s-1}^J of being in office in period $s-1$ is:

$$\begin{aligned} p_s^J &= \alpha_c p_{s-1}^J + (1 - \alpha_c)(1 - p_{s-1}^J) \\ &= IA \cdot p_{s-1}^J + 1 - \alpha_c, \end{aligned} \tag{3}$$

where $IA \equiv 2\alpha_c - 1$ denotes the incumbency advantage, the difference in probability of winning the election as incumbent party compared to the opposition party.⁹ Solving Equation (3) recursively gives:

$$p_s^J = 0.5 + IA^{s-t}(p_t^J - 0.5) \text{ for all } s \geq t \geq 0.$$

Consequently, the probability that the incumbent party I_t of period t is in office in period $s \geq t$ is:

$$p_{s-t}^I = 0.5 + 0.5 \cdot IA^{s-t}. \quad (4)$$

The probability that the opposition party O_t of period t is in office in period s is:

$$p_{s-t}^O = 0.5 - 0.5 \cdot IA^{s-t}. \quad (5)$$

The second terms in Equations (4) and (5) capture the effect of being the incumbent respectively the opposition party in period t on the probability of being in office in period s . As long as $|IA| \neq 1$, these incumbency effects are decreasing in the number of periods between s and t and increasing in the size of the incumbency advantage IA . If $IA = 1$, the incumbent party is re-elected forever, and if $IA = -1$, the two parties alternate in office after each election. If $|IA| \neq 1$, 0.5 is approximately the probability of being in office in periods that lie far ahead in the future.

The expected present discounted payoff of the incumbent party in any period t is:

$$V^I(\alpha_c, \bar{r}) = \sum_{s=t}^{\infty} \beta^{s-t} p_{s-t}^I \bar{r} = \left(\frac{1}{2(1-\beta)} + \frac{1}{2(1-\beta \cdot IA)} \right) \bar{r}. \quad (6)$$

The expected present discounted payoff of the opposition party is:

$$V^O(\alpha_c, \bar{r}) = \sum_{s=t}^{\infty} \beta^{s-t} p_{s-t}^O \bar{r} = \left(\frac{1}{2(1-\beta)} - \frac{1}{2(1-\beta \cdot IA)} \right) \bar{r}. \quad (7)$$

The difference between these two expected discounted payoffs is given by:

$$V^I(\alpha_c, \bar{r}) - V^O(\alpha_c, \bar{r}) = \frac{\bar{r}}{1 - \beta \cdot IA}. \quad (8)$$

⁹For negative values of IA , it seems more natural to use the expression "incumbency disadvantage of size $|IA|$ ". Because $0 \leq \alpha_c \leq 1$, it follows that $-1 \leq IA \leq 1$.

If there is no incumbency advantage ($IA = 0$), the difference is equal to the rent payment the incumbent party receives in the current period. If there is an incumbency advantage ($IA > 0$), the difference is larger than $\bar{r}$ because being the incumbent party also increases the likelihood of holding office in all future periods. If there is an incumbency disadvantage ($IA < 0$), the difference is smaller than $\bar{r}$ because being the incumbent party decreases the likelihood of holding office in the next period.¹⁰ Because rent-seeking is only possible while holding office, the difference in the expected discounted payoffs is always positive.

Sustainable rent-seeking thresholds The single-deviation principle states that to establish a subgame perfect equilibrium in a multi-stage game in which the overall payoffs are a discounted sum of per-period payoffs that are uniformly bounded, it is sufficient to show that no single-deviation in any subgame can make the deviating player better off.¹¹ Given the threshold $\bar{r}$, an incumbent party only needs to consider the two alternatives of either choosing the maximal amount of per-period rent-seeking of 1 or restricting itself to the threshold amount $\bar{r}$. All other levels of rent-seeking lead obviously to a lower expected discounted payoff than at least one of these two alternatives. An incumbent party that deviates in a single period and engages in the maximal amount of rent-seeking of 1 has an expected discounted future payoff of:

$$1 + \beta (\alpha_d V^I(\alpha_c, \bar{r}) + (1 - \alpha_d) V^O(\alpha_c, \bar{r})) . \quad (9)$$

$V^I(\alpha_c, \bar{r})$ can be denoted recursively:

$$V^I(\alpha_c, \bar{r}) = \bar{r} + \beta (\alpha_c V^I(\alpha_c, \bar{r}) + (1 - \alpha_c) V^O(\alpha_c, \bar{r})) . \quad (10)$$

¹⁰To be precise, it decreases the probability of being in office in all future periods for which $s - t$ is odd and increases the probability for all future periods for which $s - t$ is even. The future periods for which $s - t$ is odd are, on average, one period closer to t and therefore less heavily discounted. Moreover, the influence of being the incumbent party in period t on being the incumbent party in period s is larger for the periods which are closer to the present.

¹¹Because the maximum level of rent-seeking is constant over time, it is obvious that per-period payoffs are uniformly bounded. See Fudenberg and Tirole (1991) for a formal statement of the single-deviation principle.

It is optimal for an incumbent party to comply with the threshold amount $\bar{r}$ as long as (10) is not smaller than (9). This condition can be restated as:

$$\begin{aligned} 1 - \bar{r} &\leq \beta(\alpha_c - \alpha_d) (V^I(\alpha_c, \bar{r}) - V^O(\alpha_c, \bar{r})) \\ &= \beta \cdot EP \frac{\bar{r}}{1 - \beta \cdot IA}, \end{aligned} \quad (11)$$

where $EP \equiv \alpha_c - \alpha_d$ is the electoral punishment for not complying with the threshold $\bar{r}$. Electoral punishment can take values between 0 and 1. The left-hand side of Equation (11) is the maximal additional amount of rent-seeking that the incumbent party can engage in when deviating. The right-hand side is the expected present discounted loss of future rents that the incumbent party faces in return for deviating once. For the minimal rent-seeking threshold $\bar{r}^*$ consistent with the incumbent party complying, Equation (11) holds with equality:

$$\bar{r}^* = \frac{1 - \beta \cdot IA}{1 - \beta(IA - EP)}. \quad (12)$$

Therefore, any *CTCSP* with threshold $\bar{r} \in [\bar{r}^*, 1]$ constitutes a subgame perfect *CTCE*.

Equation (12) shows that without any electoral punishment ($EP = 0$), minimal sustainable rents are identical to the maximally possible amount of rent-seeking 1. Not surprisingly, electoral punishment is a necessary condition for electoral accountability.

Conditional on IA and EP , the *CTCE* with threshold $\bar{r}^*$ achieves the largest possible expected discounted payoff for the *RRV* that is consistent with equilibrium. Because with threshold $\bar{r}^*$ condition (11) holds with equality, the incumbent party is always indifferent between complying with the threshold or engaging in the maximally possible amount of rent-seeking 1. Therefore, its expected discounted payoff $V^I(\alpha_c, \bar{r}^*)$ is identical to $V^I(\alpha_d, \alpha_c, 1)$, the expected discounted payoff of an incumbent party that always engages in rent-seeking of 1 when in office and is always re-elected with probability α_d , while the other party is always re-elected with probability α_c .¹² Because this is an intuitive result, its formal proof is relegated to Appendix B. Because remaining in office with probability α_d and regaining it with probability $1 - \alpha_c$

¹²The formally more consequent notation would be $V^I(\alpha_c, \alpha_c, \bar{r}^*)$ instead of $V^I(\alpha_c, \bar{r}^*)$. However, if the same re-election probability α_c applies to both parties, using $V^I(\alpha_c, \bar{r}^*)$ is more convenient.

are the worst possible election odds, an incumbent party can ensure itself an expected payoff of at least $V^I(\alpha_d, \alpha_c, 1)$ by always engaging in the maximal amount of rent-seeking 1, independently of the strategies of the other players. It follows that $V^I(\alpha_c, \bar{r}^*) = V^I(\alpha_d, \alpha_c, 1)$ is the lowest expected discounted payoff for the incumbent party that is consistent with an equilibrium of the game. Also, $V^O(\alpha_c, \bar{r}^*)$ is the lowest possible expected discounted payoff for the opposition party. This follows because the opposition party's election odds are $1 - \alpha_c$ as long as it is out of office, the worst possible, and moreover, after winning an election, the former opposition party's expected discounted payoff $V^I(\alpha_c, \bar{r}^*)$ is the lowest consistent with equilibrium for an incumbent party. Therefore, both, the incumbent party and the opposition party, expect the lowest discounted rents consistent with equilibrium for their given statuses, there exists no equilibrium in which the sum of the expected discounted payoffs of the parties is lower. Because the game is zero-sum, it follows that there exists no equilibrium in which the expected discounted payoff of the *RRV* is larger.

In the literature on political accountability since Barro (1973) and Ferejohn (1986) it is a standard assumption that the electorate or a representative voter can optimize over the threshold for re-election and thus implements its preferred equilibrium of the game. Therefore, the *CTCE* with threshold $\bar{r}^*$ provides a natural starting point for the analysis. However, it turns out that there $\bar{r}^*$ is the lower bound of an interval of thresholds which are at the same time consistent with both parties complying as well as with neither party complying with the performance standard.

Equilibria with noncompliance I first discuss an equilibrium with noncompliance in which the parties never comply. Subsequently, a more sophisticated equilibrium in which the parties never comply along the equilibrium path, but switch to playing *CTCE* as soon as an incumbent party complies in a single period is discussed. Switching to *CTCE* serves as a punishment so that in the sophisticated version larger thresholds are consistent with noncompliance of the parties .

Definition 2 *Constant threshold with noncompliance strategy profile (CTNSP):*

$$\bar{r} \in [0, 1),$$

$$s_J : r_t = 1 \text{ in each } t \text{ in which } I_t = J \text{ with } J = A, B,$$

$$s_{RRV} : v_t = \begin{cases} 1 & \text{if } r_t \leq \bar{r} \\ 0 & \text{if } r_t > \bar{r} \end{cases} \text{ in all } t.$$

The case $\bar{r} = 1$ is ruled out here because it makes noncompliance impossible and does thus not constitute a real performance threshold. Once more, the strategy of the retrospective voter is a best response because, given the strategy followed by the parties, she is indifferent which party wins the election. Incumbent parties are always re-elected with probability α_d because they never comply.

Following the same steps that were used to calculate $V^I(\alpha_c, \bar{r})$ in Equation (6) and $V^I(\alpha_c, \bar{r})$ in Equation (7), the expected present discounted payoff of the incumbent party in any period t is now:

$$V^I(\alpha_d, 1) = \frac{1}{2(1 - \beta)} + \frac{1}{2(1 - \beta(IA - 2EP))}. \quad (13)$$

Here, I stick to the definition $IA \equiv 2\alpha_c - 1$ although with noncompliance the incumbency advantage is given by $2\alpha_d - 1 = IA - 2EP$.¹³ The expected present discounted payoff of the opposition party is:

$$V^O(\alpha_d, 1) = \frac{1}{2(1 - \beta)} - \frac{1}{2(1 - \beta(IA - 2EP))}. \quad (14)$$

The difference is given by:

$$V^I(\alpha_d, 1) - V^O(\alpha_d, 1) = \frac{1}{1 - \beta(IA - 2EP)}. \quad (15)$$

Once more, the single-deviation principle is applied. Noncompliance is optimal as long as:

$$1 - \bar{r} \geq \frac{\beta \cdot EP}{1 - \beta(IA - 2EP)}.$$

Here, the left-hand side is the loss from complying with the performance threshold $\bar{r}$ instead of engaging in maximal rent-seeking of 1 for one period. The right-hand-side is the additional expected discounted present payoff from winning the election with probability α_c instead of α_d .¹⁴ Solving for $\bar{r}$ gives:

$$\bar{r} \leq \frac{1 - \beta(IA - EP)}{1 - \beta(IA - 2EP)}.$$

¹³The justification is that a noncompliance equilibrium is unlikely to be played because the *RRV* chooses only thresholds for which she expects compliance by the parties. This issue is discussed in more detail later.

¹⁴As before, any other deviation from the party's strategy leads obviously to a lower expected discounted payoff than either rent-seeking of $\bar{r}$ or rent-seeking of 1.

For $\bar{r}^{**}$, the largest threshold that is consistent with a *CTNSP* constituting a *Constant threshold with noncompliance equilibrium* or *CTNE*, the inequality holds strictly:

$$\bar{r}^{**} = \frac{1 - \beta(IA - EP)}{1 - \beta(IA - 2EP)}. \quad (16)$$

While a party in opposition is always better off with the other party ignoring the threshold $\bar{r}$, an incumbent party is better off when it restricts itself to the threshold $\bar{r}$ in return for a larger chance of re-election if $\bar{r}$ is large enough. However, because an incumbent party expects to return of office sooner when it expects the other party not to comply with $\bar{r}$, it is more attractive to engage in maximal rent-seeking if the other party is doing the same. Compliance with the performance standard is a strategic complement in the sense of Bulow, Geanakoplos, and Klemperer (1985), a well-known cause of multiple equilibria. Therefore, $\bar{r}^{**} > \bar{r}^*$ as long as there is electoral punishment ($EP > 0$), and there is an interval $\bar{r} \in [\bar{r}^*, \bar{r}^{**}]$ in which *both parties never complying* and *both parties always complying* constitute two different equilibria with the same performance standard. This is an important consequence of the setup with two parties that interchange in office, and, to the best of my knowledge, new to the literature on political accountability. Not surprisingly, for a given threshold $\bar{r} \in [\bar{r}^*, \bar{r}^{**}]$, the incumbent party as well as the opposition party are better off with the equilibrium in which neither party complies. This intuitive result is shown formally at the end of the next subsection.

A sophisticated equilibrium with noncompliance Now I consider the possibility that the parties punish each other for complying.¹⁵ The only possibility for a party to punish the other party is to start complying itself after the other party has deviated from noncompliance. In other words, the threat of the worst equilibrium from the perspective of the parties, the *CTCE*, is used to sustain an equilibrium with maximal rent-seeking in every period. The parties are colluding in their noncompliance with the threshold and punish each other for deviations from this collusion. With the *CTCE* as punishment for deviating incumbent parties, it is possible to sustain

¹⁵Usually players are punished for noncompliance and not for compliance. However, in a non-compliance equilibrium compliance with the *RRV* is akin to cheating the party that is out of office. There is some similarity with repeated oligopoly games with collusion in which firms try to restrict output and increase prices relative to the competitive level. There, deviating and cheating the colluding competitors is also socially desirable, exactly as restricting rent-seeking in the model discussed here.

maximal rent-seeking for an interval of thresholds $\bar{r} > \bar{r}^{**}$ for which noncompliance is not sustainable as a *CTNE*.

Definition 3 *Sophisticated constant threshold with noncompliance strategy profile (SCTNSP):*

$$\begin{aligned} \bar{r} &\in [0, 1), \\ s_J : r_t &= \begin{cases} 1 & \text{if } r_s > \bar{r} \text{ in every period } s < t \\ r_t = \bar{r} & \text{if } r_s \leq \bar{r} \text{ in any period } s < t \end{cases} \quad \text{in each } t \text{ in which } I_t = J \text{ with} \\ J &= A, B, \\ s_{RRV} : v_t &= \begin{cases} 1 & \text{if } r_t \leq \bar{r} \\ 0 & \text{if } r_t > \bar{r} \end{cases} \quad \text{in all } t. \end{aligned}$$

From the moment an incumbent party deviates by complying with $\bar{r}$, both parties always comply. Consequently, the punishment subgame with both parties complying is consistent with subgame perfect equilibrium as long as $\bar{r} \geq \bar{r}^*$, the same condition as for the *CTCE*.

An incumbent party that deviates from the main phase of the game in which both parties play noncompliance and instead complies with $\bar{r}$ switches the play of the game to the *CTCE*. Consequently, such a deviating incumbent party has an expected discounted payoff of $V^I(\alpha_c, \bar{r})$, the same as in the *CTCE*. If an incumbent party stays on the equilibrium path, its expected present discounted payoff is $V^I(\alpha_d, 1)$. Therefore, a *SCTNSP* constitute a subgame perfect *Sophisticated constant threshold with noncompliance equilibrium* or *SCTNE* for all values $\bar{r} \geq \bar{r}^*$ for which:

$$V^I(\alpha_d, 1) \geq V^I(\alpha_c, \bar{r}). \quad (17)$$

Let $\bar{r}^{***}$ be implicitly defined by $V^I(\alpha_d, 1) = V^I(\alpha_c, \bar{r}^{***})$. Solving for $\bar{r}^{***}$ gives:

$$\bar{r}^{***} = \frac{\frac{1}{2(1-\beta)} + \frac{1}{2(1-\beta(IA-2EP))}}{\frac{1}{2(1-\beta)} + \frac{1}{2(1-\beta \cdot IA)}} = \frac{2 - \beta(1 + IA - 2EP)}{1 - \beta(IA - 2EP)} \frac{1 - \beta \cdot IA}{2 - \beta(1 + IA)}. \quad (18)$$

Because $V^I(\alpha_c, \bar{r})$ is strictly increasing in $\bar{r}$, inequality (17) holds for all $\bar{r} \leq \bar{r}^{***}$ and $\bar{r}^{***}$ is the largest threshold amount for which *SCTNSP* constitutes a *SCTNE*. Comparing $\bar{r}^*$, $\bar{r}^{**}$ and $\bar{r}^{***}$, given in equations (12), (16) and (18), shows that $EP > 0$ implies $0 < \bar{r}^* < \bar{r}^{**} \leq \bar{r}^{***} < 1$, while $EP = 0$ implies $\bar{r}^* = \bar{r}^{**} = \bar{r}^{***} = 1$, confirming once more that there is no accountability without electoral punishment. If $IA = 1$, an incumbent party which always complies never loses office. In this

case, a deviation from the *SCTNE* cannot be punished with compliance by the other party because it never gains office. Therefore, the incentives for noncompliance in the *SCTNE* are not stronger than in the *CTNE* and $\bar{r}^{**} = \bar{r}^{***}$. In all cases with $EP > 0$ and $IA < 1$, the incentives for noncompliance are stronger in the *SCTNE* and thus $\bar{r}^{**} < \bar{r}^{***}$.¹⁶ From condition (17), the definition of $\bar{r}^{***}$ and $\bar{r}^{**} \leq \bar{r}^{***}$ follows that the incumbent party is strictly better off with the *CTNE* or the *SCTNE* compared to the *CTCE* for all $\bar{r} < \bar{r}^{***}$ for which more than one equilibrium exists.

Summary - Equilibria with constant thresholds *CTNE* is the only equilibrium for $\bar{r} < \bar{r}^*$ because for such low thresholds the additional chance of re-election is not sufficient to induce compliance, independently of the strategy of the other party. If $\bar{r} \in [\bar{r}^*, \bar{r}^{***}]$, either both parties complying or neither party complying with the threshold are both consistent with equilibrium. While for $\bar{r} \in [\bar{r}^*, \bar{r}^{**}]$ noncompliance is a best response if the other party also always plays noncompliance and thus all three types of equilibrium with constant performance standard exist, for $\bar{r} \in (\bar{r}^{**}, \bar{r}^{***}]$ noncompliance of incumbent parties is only sustainable in a *SCTNE*, while a *CTNSP* is not consistent with equilibrium. For $\bar{r} > \bar{r}^{***}$, only the *CTCE* constitutes an equilibrium.¹⁷

A forward induction argument Within the interval $\bar{r} \in [\bar{r}^*, \bar{r}^{***})$, the *CTCE* is not robust against the following forward induction argument: Assume the incumbent party deviates and does not comply in the period(s) immediately before it loses office. Even without communication, and communication between parties is certainly possible, the new incumbent party has reason to believe that this was not a single incidence of deviation. It seems plausible to interpret the deviation as also communicating a message along the following lines: *Our party is now engaging in maximal rent-seeking. Maximal rent-seeking is only rational for us if we believe that you will also play noncompliance from now on. Given the threshold $\bar{r} \in [\bar{r}^*, \bar{r}^{***})$, it is rational for you to do so. Otherwise, we will also cease doing so, and we are both worse off*

¹⁶Formally, $\bar{r}^{**} < \bar{r}^{***}$ follows from inspecting $\bar{r}^{***} - \bar{r}^{**} = \frac{\beta^2(1-IA)EP}{(2-\beta(1+IA))(1+\beta(2EP-IA))} \geq 0$ which holds with equality only if $IA = 1$ or $EP = 0$ and otherwise holds strictly.

¹⁷To avoid parties that are indifferent between compliance and noncompliance along the equilibrium path, the *RRV* can use slightly larger thresholds ($\bar{r}^* + \varepsilon$, $\bar{r}^{**} + \varepsilon$ or $\bar{r}^{***} + \varepsilon$ with $\varepsilon > 0$ and very small) instead of $\bar{r}^*$, $\bar{r}^{**}$ and $\bar{r}^{***}$.

with the resumption of the *CTCE*. In other words, we just started playing *SCTNE*.¹⁸ The argument is especially strong as long as $\bar{r} < \bar{r}^{**}$ because in this case noncompliance is a best response against noncompliance even without the threat of a *CTCE* as a punishment.

Therefore, for a constant threshold $\bar{r} \in [\bar{r}^*, \bar{r}^{***})$ and definitely for $\bar{r} \in [\bar{r}^*, \bar{r}^{**}]$, it seems reasonable to expect that after an incumbent party once does not comply with the performance threshold, both parties engage in maximal rent-seeking forever. Loosely speaking, the parties switch from a *CTCE* to *CTNE* or *SCTNE*. However, if the incumbent party of the first period foresees this, it deviates from a *CTCE* right away. Therefore, either *CTNE* or *SCTNE* is likely to be implemented from the very first period and compliance of the parties is not achievable with any constant threshold $\bar{r} < \bar{r}^{***}$.

The optimal constant threshold for the representative retrospective voter If the thresholds are not due to social norms, but, as usually assumed in the literature, determined deliberately by the retrospective voters to hold the parties accountable, a rational *RRV* chooses the lowest constant threshold $\bar{r}$ for which she expects compliance of the parties. Consequently, a *CTCE* is the expected equilibrium if the *RRV* implements a constant performance standard. However, given the forward induction argument stated before, the performance standard in this *CTCE* would be $\bar{r}^{***}$ or perhaps $\bar{r}^{**}$, but not the most stringent $\bar{r}^*$. If a *CTCE* is played, the incumbent party is always re-elected with probability α_c . The expectation of a *CTCE* therefore justifies the definition of the incumbency advantage as $IA \equiv 2\alpha_c - 1$. However, as is discussed next, the *RRV* can achieve $\bar{r}^*$ along the equilibrium path if thresholds conditional on the history of the game can be implemented.

An equilibrium with changing thresholds The *RRV* can punish a deviating party by changing the threshold if the deviating party loses office in the election following the deviation. By changing the threshold, she can ensure that the deviating party regains office only with the worst possible odds of $1 - \alpha_c$ per period independently of the strategy of the other party. Specifically, the *RRV* temporarily lowers

¹⁸While this argument is invoking a form of forward induction, I have been unable to find any other application of forward induction to infinite stage games. The concept of forward induction is explained, for example, in Mas-Colell, Whinston, and Green (1995).

the performance standard to $\bar{r}_t = 1$ for as long as the non-deviating party stays in office.

Definition 4 *Variable threshold with compliance equilibrium (VTCE):*

$$\bar{r} \in [\bar{r}^*, 1],$$

$$\bar{r}_0 = \bar{r}$$

$$\bar{r}_t = \begin{cases} 1 & \text{if } (I_t \neq I_{t-1} \wedge r_{t-1} > \bar{r}_t) \\ 1 & \text{if } (I_t = I_{t-1} \wedge \bar{r}_{t-1} = 1) \\ \bar{r} & \text{otherwise} \end{cases} \quad \text{in all } t \geq 1,$$

$$s_J : r_t = \bar{r}_t \text{ in each } t \text{ in which } I_t = J \text{ with } J = A, B,$$

$$s_{RRV} : v_t = \begin{cases} 1 & \text{if } r_t \leq \bar{r} \\ 0 & \text{if } r_t > \bar{r} \end{cases}.$$

As before, the strategy of the *RRV* does not influence her payoff and is thus a best response. The strategy of the parties is to always comply with $\bar{r}_t$. Consequently, along the equilibrium path of the game, they are re-elected with probability α_c . Whenever $r_t = 1$, engaging in maximal rent-seeking does not change any expected future payoffs and is thus optimal. As long as $\bar{r}_t = \bar{r}$, it is straightforward to confirm that incumbent parties face the same trade-offs as in a *CTCE* and thus compliance is consistent with equilibrium because $\bar{r} \geq r^*$. The important difference to the *CTCE* is that now, as long as $\bar{r}_t = \bar{r}$, complying is optimal independently of the strategy of the other party. The reason is that because of the changing threshold, noncompliance as a response to noncompliance ceases to be a possibility. However, without noncompliance as a response to noncompliance, the possibility of equilibria with both parties playing noncompliance does not arise, and the forward induction argument that made the *CTCE* with threshold $\bar{r}^*$ implausible does not apply. Hence, increasing the threshold for the other party in response to a deviation provides a punishment for noncomplying parties. For a sophisticated *RRV*, the *VTCE* with $\bar{r} = \bar{r}^*$ (respectively $\bar{r}^* + \varepsilon$, with $\varepsilon > 0$ and very small) is a robust way to implement the rent-seeking threshold $\bar{r}^*$ which has already been shown to be the most stringent performance standard that is consistent with compliance in equilibrium. Because the threshold becomes only less stringent than $\bar{r}^*$ off the equilibrium path, the use of the history-dependent threshold imposes no cost on the *RRV*. Equilibria with changing thresholds are not new to the literature on political accountability but can be found in an unpublished paper by Schwabe (2011) in which it is shown to be optimal to evaluate politicians of different types by different performance standards.

2.2 Comparative statics

First, results for the most stringent threshold $\bar{r}^*$ are derived and then it is confirmed that the comparative statics of the model are qualitatively the same for $\bar{r}^{**}$ and $\bar{r}^{***}$. The most interesting result is the effect of incumbency advantage on the minimal sustainable amount of rent-seeking $\bar{r}^*$ given in Equation (12):

$$\frac{\partial \bar{r}^*}{\partial IA} = -\frac{\beta^2 \cdot EP}{(1 - \beta(IA - EP))^2} \leq 0, \quad (19)$$

with strict inequality if $EP > 0$ and equality if $EP = 0$. Therefore, a larger incumbency advantage increases accountability when the amount of electoral punishment is positive. The reason for this result is that the more likely the respective incumbent party is to be re-elected in future periods, the more important it is to win the next election. As a consequence, with a larger incumbency advantage, the incumbent party is willing to forgo a larger amount of rent-seeking opportunities in return for avoiding electoral punishment, and a lower rent-seeking threshold is consistent with equilibrium.

Sustainable minimal rents are also decreasing in EP :

$$\frac{\partial \bar{r}^*}{\partial EP} = -\frac{\beta(1 - \beta \cdot IA)}{(1 - \beta(IA - EP))^2} < 0, \quad (20)$$

where the inequality follows from $IA \leq 1$ and $\beta < 1$.

While electoral punishment is a necessary condition for the incumbency advantage to influence accountability, the size of the incumbency advantage also provides an upper bound for the electoral punishment because $EP \leq \frac{1+IA}{2}$. The lowest possible value of r^* is reached when $IA = EP = 1$ and thus $\bar{r}^* = 1 - \beta$.

Now I confirm that qualitatively, the comparative statics are the same for $\bar{r}^{**}$ and $\bar{r}^{***}$ given in Equations (16) and (18). Once more, an increase in the size of the incumbency advantage allows for stricter performance standards:

$$\begin{aligned} \frac{\partial \bar{r}^{**}}{\partial IA} &= \frac{-\beta^2 \cdot EP}{(1 - \beta(IA - 2EP))^2} \leq 0, \\ \frac{\partial \bar{r}^{***}}{\partial IA} &= -\frac{2\beta^2(1 - \beta)(3 - \beta(1 + 2(IA - EP)))EP}{(2 - \beta(1 + IA))^2(1 + \beta(2EP - IA))^2} \leq 0. \end{aligned}$$

Again, both inequalities hold strictly if $EP > 0$ while $EP = 0$ implies $\frac{\partial \bar{r}^{**}}{\partial IA} = \frac{\partial \bar{r}^{***}}{\partial IA} = 0$. An increase in electoral punishment also increases accountability if the rent-seeking thresholds $\bar{r}^{**}$ or $\bar{r}^{***}$ are implemented by the *RRV*:

$$\begin{aligned}\frac{\partial \bar{r}^{**}}{\partial EP} &= -\beta \frac{1 - \beta \cdot IA}{(1 - \beta(IA - 2EP))^2} < 0, \\ \frac{\partial \bar{r}^{***}}{\partial EP} &= -\frac{2\beta(1 - \beta)(1 - \beta \cdot IA)}{(2 - \beta(1 + IA))(2\beta \cdot EP - \beta \cdot IA + 1)^2} < 0.\end{aligned}$$

Thus, the qualitative comparative statics are the same for all three discussed performance standards.

3 The electorate

The model analyzed so far allows for different specifications of the voters and their motives, and, with some adjustments, is probably consistent with political systems in which accountability is achieved by other means than elections.¹⁹ Nonetheless, it is useful to extend the analysis so far with a more explicit model of the electorate. In contrast to most of the literature, which focuses on electorates that are either purely forward-looking or purely backward-looking at the ballot box, here retrospective voters vote according to the amount of rent-seeking they observe, while the prospective voters cast their ballots according to their probabilistic party preferences. The source of these preferences is not made explicit in the model, but they can, for example, depend on the policy platforms, the candidates, the influence of the media or the success of the election campaigns of the parties. Most likely, they are due to a combination of all these and other factors.

One additional result derived in this section is that a considerable number of retrospective voters comes at a cost: The larger the share of retrospective voters, the more often the party which is preferred by the majority of the prospective voters loses an election.

That some degree incumbency advantage is a natural feature of models of political accountability even when there are no structural asymmetries between incumbents and opposition has already been pointed out by Austen-Smith and Banks (1989). This

¹⁹Obviously, this can only be the case for political systems in which politicians are more likely to lose office if their accomplishments are not sufficient to reach some form of performance standard.

form of incumbency advantage can thus be called *accountability-driven* incumbency advantage. The model also allows for *preference-driven* incumbency advantage that occurs whenever the random distribution of party preferences is biased in favor of the incumbent party. This bias could, for example, be due to the fact that the party in office can use the resources of government to convince parts of the electorate of its policies. Both sources of incumbency advantage can coincide and reinforce each other. The effect of the incumbency advantage on accountability is independent of its cause.

The rules of the elections Except for the election stage, the model discussed here is identical to the one described in Section 2. Instead of a single *RRV*, there is now a continuum of voters. The incumbent party wins the election if it achieves a share of at least 0.5 of the votes. Abstentions are not possible. The retrospective voters coordinate on a threshold amount $\bar{r}_t$ and in every period t all retrospective voters vote for the incumbent party as long as $r_t \leq \bar{r}_t$ and against it whenever $r_t > \bar{r}_t$. Therefore, the *RRV* introduced in Section 2 is representative for this group of voters.

None of the voters is pivotal As a consequence of the continuum assumption, no single vote is ever pivotal for an election outcome. Nonetheless, given the voting decisions described in detail below, none of the voters would regret her vote if it were pivotal. Because the parties face the same incentives and constraints as in Section 2, the equilibrium analysis there is robust to the extension of the model presented here.

The distribution of the voters Let $s_r \in [0, 0.5]$ denote the share of retrospective voters and $s_p = 1 - s_r$ the share of prospective voters who vote according to their random party preferences.²⁰ The share of voters for the incumbent party among the voters with party preferences is determined in every election period t when nature draws their share ι_t . The outcome of this random process becomes only observable after the election in period t has taken place. The *PDF* and *CDF* of these independent

²⁰Any $s_r \geq 0.5$ implies $\alpha_c = 1$ and $\alpha_d = 0$. The resulting levels of incumbency advantage, electoral punishment and accountability are the same for all $s_r \geq 0.5$ and can be calculated by using $s_r = 0.5$ in the equation of interest.

and identically distributed random variables ι_t are in all periods t given by:

$$f(\iota) = \begin{cases} 0 & \text{if } \iota < 0 \\ 1 + b(2\iota - 1) & \text{if } 0 \leq \iota \leq 1 \\ 0 & \text{if } \iota > 1 \end{cases}, \quad F(\iota) = \begin{cases} 0 & \text{if } \iota < 0 \\ \iota + b(\iota^2 - \iota) & \text{if } 0 \leq \iota \leq 1 \\ 1 & \text{if } \iota > 1 \end{cases},$$

where $b \in [-1, 1]$ is the bias in favor of the incumbent party. If there is no bias, $b = 0$ and ι_t is uniformly distributed on the interval $[0, 1]$. The variable b allows for a *preference-driven* source of incumbency advantage by moving some of the density of the distribution of ι_t from a low vote share in favor of the incumbent to a large vote share favor of the incumbent if $b > 0$.²¹

The probabilities of re-election If the incumbent party complies in period t , it wins if and only if the sum of the retrospective voters and its supporter among the prospective voters constitute a majority. This is the case if and only if $s_r + \iota_t \cdot s_p \geq 0.5$ or:

$$\iota_t \geq \frac{0.5 - s_r}{s_p} = \frac{1 - 2s_r}{2 - 2s_r}.$$

Given the distribution of ι_t , the probability that this inequality holds and the non-deviating incumbent party wins is:

$$\alpha_c = 1 - F\left(\frac{1 - 2s_r}{2 - 2s_r}\right) = \frac{1}{2(1 - s_r)} + \frac{b(1 - 2s_r)}{4(1 - s_r)^2}.$$

The probability of victory is 1 if $s_r = 0.5$ and the retrospective voters alone decide the election. If $s_r = 0$, there are no retrospective voters, $\alpha_c = \frac{2+b}{4}$ and there is only an incumbency advantage if $b > 0$.

Only the incumbent party can win without the support of the majority of the prospective voters. The probability that this happens in any given election is:

$$\int_{0.5-s_r}^{0.5} f(\iota) d\iota = s_r - b \cdot s_r^2.$$

Therefore, the probability that an incumbent party is re-elected although the majority of prospective voters prefers the opposition party is increasing in the share of retrospective voters and decreasing in the bias in favor of the incumbent party. The

²¹In case $b < 0$, the incumbency advantage is reduced instead.

average size of the majority, short AM , in favor of the winner of the election among the prospective voter is given by:

$$AM = \int_{0.5-s_r}^1 (2\iota - 1)f(\iota)d\iota + \int_0^{0.5-s_r} (1 - 2\iota)f(\iota)d\iota = 0.5 - 2s_r^2 + \frac{8}{3}bs_r^3.$$

For a considerable bias against the incumbent party (negative b), AM can take negative values. The average size of the majority AM is decreasing in the share of retrospective voters:

$$\frac{\partial AM}{\partial s_r} = 4s_r(2bs_r - 1) \leq 0,$$

which holds with equality if $b = 1$ and $s_r = 0.5$ and otherwise strictly. Therefore, a larger share of retrospective voters decreases the influence of the prospective voters on the outcome of the elections. While the welfare consequences of this reduced influence are not explicitly modelled, these results indicate that a large share of retrospective voters comes at a cost. A noncomplying incumbent party is only supported by prospective voters and wins in any period t if and only if $\iota_t \cdot s_p \geq 0.5$ or:

$$\iota_t \geq \frac{1}{2s_p} = \frac{1}{2(1-s_r)}.$$

It follows that:

$$\alpha_d = 1 - F\left(\frac{1}{2(1-s_r)}\right) = 1 - \frac{1}{2(1-s_r)} + \frac{b(1-2s_r)}{4(1-s_r)^2}.$$

Using the definitions of the incumbency advantage and the electoral punishment, it is now possible to calculate how they are determined by the variables describing the electorate:

$$\begin{aligned} IA &= \frac{s_r}{1-s_r} + \frac{b(1-2s_r)}{2(1-s_r)^2}, \\ EP &= \frac{s_r}{1-s_r}. \end{aligned} \tag{21}$$

There is only electoral punishment when there are retrospective voters ($s_r > 0$). If there is no electoral bias in favor of one of the parties ($b = 0$), it turns out that $IA = EP = \frac{s_r}{1-s_r}$. The reason is that IA and EP are both given by the difference between the probability of winning with and without the support of the retrospective

voters. Because electoral punishment depends only on the share of retrospective voters, estimates of their share in the electorate could be the foundation for the estimate of the electoral punishment.

The incumbency advantage is increasing in the share of retrospective voters and increasing in the electoral bias for the incumbent party:

$$\begin{aligned}\frac{\partial IA}{\partial s_r} &= \frac{1}{(1-s_r)^2} - b \frac{s_r}{(1-s_r)^3} \geq 0, \\ \frac{\partial IA}{\partial b} &= \frac{1-2s_r}{2(1-s_r)^2} > 0.\end{aligned}\tag{22}$$

If $s_r = 0.5$ and $b = 1$, then $\frac{\partial IA}{\partial s_r} = 0$.²² All other values of s_r and b imply $\frac{\partial IA}{\partial s_r} > 0$. The incumbency advantage increases with the share of retrospective voters because in equilibrium all of them support the incumbent party. Electoral punishment is independent of the bias in favor of the incumbent party and increases in the share of retrospective voters:

$$\begin{aligned}\frac{\partial EP}{\partial b} &= 0, \\ \frac{\partial EP}{\partial s_r} &= \frac{1}{(1-s_r)^2} > 0.\end{aligned}\tag{23}$$

Using the results derived in Section 2.2, an informal application of the chain rule implies that an increase in b , the bias in favor of the incumbent party, increases accountability because it increases the size of the incumbency advantage. An increase in the share of retrospective voters increases both the incumbency advantage and the electoral punishment and therefore also leads to an increase in accountability. The next subsection confirms these results formally.

Rent-seeking and the electorate Substituting for IA and EP by s_r and b in Equation (12) shows how the minimal sustainable amount of rent-seeking depends on the variables b and s_r which describe the electorate:

$$\bar{r}^* = \frac{1 - \beta \cdot IA}{1 - \beta(IA - EP)} = 1 - \frac{2\beta(1-s_r)s_r}{2(1-s_r)^2 - \beta \cdot b(1-2s_r)}.\tag{24}$$

²²Because $s_r \leq 0.5$ by assumption, if $s_r = 0.5$ and $b = 1$ strictly speaking only the left partial derivative of IA with respect to s_r exists.

It is immediate that without retrospective voters ($s_r = 0$), $\bar{r}^* = 1$ and there is no accountability. With $s_r = 0.5$, the retrospective voter can guarantee a complying incumbent party to stay in office and $\bar{r}^* = 1 - \beta$, the lowest possible threshold. I confirm the results already established informally by calculating the partial derivatives of $\bar{r}^*$ with respect to the two exogenous variables b and s_r :

$$\frac{\partial \bar{r}^*}{\partial b} = -\frac{2\beta^2(1-s_r)(1-2s_r)s_r}{(2(1-s_r)^2 - \beta b(1-2s_r))^2} \leq 0,$$

with strict inequality if $s_r > 0$. Because the electoral bias in favor of the incumbent party does not influence the electoral punishment, the influence of b on $\bar{r}^*$ is entirely due to the effect on the incumbency advantage.

$$\frac{\partial \bar{r}^*}{\partial s_r} = \frac{-4\beta(1-s_r)^2 - 2\beta \cdot b(2\beta(1-s_r)s_r - \beta)}{(2(1-s_r)^2 - \beta b(1-2s_r))^2} < 0.$$

Here, the effect on accountability is partly due to the change in incumbency advantage and partly due to the change in electoral punishment. Applying the chain rule to Equation (12) makes it possible to distinguish the impact of the two channels by which the share of retrospective voters impacts accountability. This is relegated to Appendix C.

4 Conclusion

Many readers might perceive the finding that a larger incumbency advantage reduces the amount of rent-seeking as counterintuitive because a high likelihood of re-election for the incumbent party seems to indicate a lack of political competition. However, the closeness of an election provides at best a rough indicator of its competitiveness. It can be compared to the use of market shares as a first indicator in industrial economics. And just as microeconomists have developed more advanced measures of economic competitiveness based on economic theory, there is a similar need for such theory-based indicators for elections. My analysis suggests electoral punishment as one such indicator because it corresponds to the trade-offs parties in office are facing. Unfortunately, in empirical work, estimating the electoral punishment is more challenging than measuring the incumbency advantage because electoral punishment can never be observed in the equilibrium of the game. This might be the reason why

applied research has, so far, focused on the incumbency advantage instead of some alternative indicators.

The probabilistic voting model presented here allows two different groups of voters that use different voting strategies. In future research, this framework can be extended to additional groups. For example, the introduction of partisan voters could allow for party specific incumbency advantage and a detailed analysis of the effects gerrymandering, the practice of manipulating district boundaries to improve the electoral prospects of a party which is common in the US. While there is an enormous literature on the issue,²³ the effects of gerrymandering on the incentives of incumbent politicians have so far not received much attention. However, this paper has shown that incentives matter for incumbent politicians as much as equilibrium re-election rates and a party that has the opportunity to engage in a gerrymander is likely to have additional objectives besides increasing the number of districts it is likely to win. For example, it might want to reduce the accountability of its candidates or ensure that it can implement its favorite policies.

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²³For a recent overview, see Holden (2016).

Appendix

Appendix A - German federal elections and chancellors since 1949 Table 1 contains the results of the German federal elections since 1949. The chancellor is elected by the German parliament (Bundestag) whose members are elected in a general election. While the chancellor is not elected directly, her position is the most important outcome of German federal elections, and the party of the chancellor is usually considered to be the winner of the election. The early election of 1983 is problematic because it was held shortly after the FDP, a smaller party that used to be the coalition partner of the SPD, switched partners and instead formed a coalition with the CDU. This resulted in chancellor Helmut Schmidt (SPD) losing office to Helmut Kohl (CDU) without a general election taking place. Because this happened only a few months before the early election of 1983, at least from the perspective of retrospective voters, the SPD rather than the CDU should be seen as the incumbent party in 1983. However, this has only a marginal effect on the incumbency advantage in the sample: With the SPD as the incumbent party in 1983, the incumbent party loses the chancellorship after 4 of 18 elections, still less than 1/4, and the SPD wins 4 out of 6 elections in which it provides the incumbent chancellor. 1982 has so far been the only time that the chancellorship switched between SPD and CDU between general elections.

"Insert Table 1 here"

Year of election	Party affiliation and name of Chancellor immediately before the election	Party affiliation and name of Chancellor immediately after the election	Incumbent party victory?
1949	None - first general election after war	CDU Konrad Adenauer	No incumbent party
1953	CDU Adenauer	CDU Konrad Adenauer	yes
1957	CDU Adenauer	CDU Konrad Adenauer	yes
1961	CDU Adenauer	CDU Konrad Adenauer	yes
1965	CDU Ludwig Erhard	CDU Ludwig Erhard	yes
1969	CDU Kurt Georg Kiesinger	SPD Willy Brandt	no
1972	SPD Willy Brandt	SPD Willy Brandt	yes
1976	SPD Helmut Schmidt	SPD Helmut Schmidt	yes
1980	SPD Helmut Schmidt	SPD Helmut Schmidt	yes
1983	CDU Helmut Kohl	CDU Helmut Kohl	Formally yes. However, Kohl (CDU) had only become chancellor a few months before the election when he succeeded Schmidt (SPD). Consequently, retrospective voters could only evaluate the tenure of Schmidt since 1980.
1987	CDU Helmut Kohl	CDU Helmut Kohl	yes
1990	CDU Helmut Kohl	CDU Helmut Kohl	yes
1994	CDU Helmut Kohl	CDU Helmut Kohl	yes
1998	CDU Helmut Kohl	SPD Gerhard Schröder	no
2002	SPD Gerhard Schröder	SPD Gerhard Schröder	yes
2005	SPD Gerhard Schröder	CDU Angela Merkel	no
2009	CDU Angela Merkel	CDU Angela Merkel	yes
2013	CDU Angela Merkel	CDU Angela Merkel	yes
2017	CDU Angela Merkel	CDU Angela Merkel	yes

Table 1 - German election results since 1949

Appendix B - The lowest expected discounted payoff consistent with equilibrium It is shown that $V^I(\alpha_c, \bar{r}^*) = V^I(\alpha_d, \alpha_c, 1)$ where $V^I(\alpha_d, \alpha_c, 1)$ denotes the expected discounted payoff of an incumbent party that always engages in maximal rent-seeking of 1 and is being re-elected with probability α_d , while the other party is always re-elected with probability α_c .

Proof. Using (12) to substitute for r^* in (6) shows that $V^I(\alpha_c, \bar{r}^*) = \frac{1}{1-\beta} \cdot \frac{1-\beta\alpha_c}{1-\beta(\alpha_c+\alpha_d-1)}$. If party K is re-elected with probability α_c and the other party with probability α_d , the probability p_s^K of party K being in office in period s given a probability p_{s-1}^K of being in office in period $s-1$ is:

$$p_s^K = \alpha_d p_{s-1}^K + (1 - \alpha_c)(1 - p_{s-1}^K) = (\alpha_c + \alpha_d - 1)p_{s-1}^K + (1 - \alpha_c).$$

Solving recursively gives:

$$p_s^K = \frac{1 - \alpha_c}{(1 - \alpha_c) + (1 - \alpha_d)} + (\alpha_c + \alpha_d - 1)^{s-t} \left(p_t^K - \frac{1 - \alpha_c}{(1 - \alpha_c) + (1 - \alpha_d)} \right) \text{ for all } s \geq t \geq 0.$$

The expected discounted payoffs of party K in any period in which it is the incumbent party is:

$$\begin{aligned} V^I(\alpha_d, \alpha_c, 1) &= \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{1 - \alpha_c}{(1 - \alpha_c) + (1 - \alpha_d)} + (\alpha_c + \alpha_d - 1)^{s-t} \left(1 - \frac{1 - \alpha_c}{(1 - \alpha_c) + (1 - \alpha_d)} \right) \right) \\ &= \frac{1 - \alpha_c}{(1 - \beta)((1 - \alpha_c) + (1 - \alpha_d))} + \frac{1 - \alpha_d}{(1 - \beta(\alpha_c + \alpha_d - 1))((1 - \alpha_c) + (1 - \alpha_d))} \\ &= \frac{1}{1 - \beta} \cdot \frac{1 - \beta\alpha_c}{1 - \beta(\alpha_c + \alpha_d - 1)} = V^I(\alpha_c, \bar{r}^*). \end{aligned}$$

■

Appendix C - The transmission of the share of retrospective voters on accountability Applying the chain rule to Equation (12) makes it possible to distinguish the impact the two channels by which the share of retrospective voters impacts accountability. Substituting for IA and EP in equations (19) and (20) using Equation (21), $\frac{\partial \bar{r}^*}{\partial IA}$ and $\frac{\partial \bar{r}^*}{\partial EP}$ are denoted as functions of s_r and b :

$$\begin{aligned}
\frac{\partial \bar{r}^*}{\partial s_r} &= \frac{\partial \bar{r}^*}{\partial IA} \cdot \frac{\partial IA}{\partial s_r} + \frac{\partial \bar{r}^*}{\partial EP} \cdot \frac{\partial EP}{\partial s_r} \\
&= -\frac{4\beta^2(1-s_r)^3 s_r}{(2(1-s_r)^2 - \beta b(1-2s_r))^2} \cdot \left(\frac{1}{(1-s_r)^2} - b \frac{s_r}{(1-s_r)^3} \right) \\
&\quad - \frac{2\beta(1-s_r)^2(2(1-s_r)^2 - 2\beta(1-s_r)s_r - \beta b(1-2s_r))}{(2(1-s_r)^2 - \beta b(1-2s_r))^2} \cdot \frac{1}{(1-s_r)^2} \\
&= \frac{-4\beta(1-s_r)^2 - 2\beta \cdot b(2\beta(1-s_r)s_r - \beta)}{(2(1-s_r)^2 - \beta b(1-2s_r))^2}
\end{aligned}$$

Which of the two channels is more important depends on the exact values of s_r , b and β .

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