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Efficient Dynamic Yield Curve Estimation in Emerging Financial Markets

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Abstract

The current state-of-the-art estimation of yield curves relies on the dynamic state space version of the Nelson and Siegel (1987) model proposed in the seminal paper by Diebold et al. (2006). However, things become difficult when applying their approach to emerging economies with less frequently bond issuance and more sparse maturity available. Therefore, the traditional state space representation, which requires dense and fixed grids of maturities, may not be possible. One remedy is to use the traditional Nelson and Siegel (1987) OLS estimation instead, though it sacrifices efficiency by ignoring the time dimension. We propose a simple augmentation of the Diebold et al. (2006) framework, which is more efficient than OLS estimation as it allows exploiting information from all available bonds and the time dependency of yields. We demonstrate the efficiency gains generated by our method in five case studies for major emerging economies including four of the BRICS.

Keywords: Yield curve, dynamic modeling, state space model, efficiency, BRICS

JEL: E52, E43

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1 Introduction

In this paper, we present a refined method for the estimation of yield curves in a dynamic framework with sparse data. The method is tailored to emerging economies, with often young and still developing financial markets. Over the past decades, the financial markets of many emerging countries experienced unprecedented growth in both depth and size. Much like their counterparts in the old industrialized world, their (government bond) yield curves have received increasing attention, both due to their importance in explaining macroeconomic dynamics and particularly due to their performance in macroeconomic forecasts.

Nel (1996) studies the correlation between the interest rate term structure and real economic variables in South Africa and compares South Africa case with G7 ones. Besides, Khomo and Aziakpono (2007) forecasts the recession of South Africa with many indicators including the interest rate term structure. Mehl (2006) examines the power of yield curve slope in forecasting emerging economies. Vicente and Tabak (2008) estimates the yield curves of Brazil by referring to many standard methods available then and concludes that the Diebold-Li method is superior to all other candidates. Rezende and Ferreira (2013) works on the same dataset but with a broader range of maturities and compares the forecasting performance of various Nelson-Siegel extended models.

All of this growing literature is plagued by massive problems due to the sparsity of the data. At this point, most of the literature on developed countries exploits information on the entire yield curve when conducting analysis and forecasting. This is usually done through an augmented Diebold et al. (2006) model, where the yield
curve is modeled dynamically based on three underlying latent factors corresponding to the factors proposed by Nelson and Siegel (1987) to describe a yield curve. For emerging markets this has often been impossible. Just looking at the previously mentioned papers, we find that almost all of them had to find workarounds to deal with missing yield curve data. Nel (1996), Khomo and Aziakpono (2007), and Mehl (2006) all have to rely on the spread between the default-free long term and the short term bond interest rate, which is merely a simple proxy for the slope of the yield curve. Mehl (2006) explicitly claims that the scarcity of emerging markets forces them to take detours to generate the term structure variables. Vicente and Tabak (2008) only cover maturities up to 12-month, i.e. they do not even include actual long run interest rates, and have to rely on interest rate swaps.

The data problems are mostly due to emerging countries not issuing bonds as often as in their developed counterparts, and therefore the time to maturity of outstanding bonds is sparse, which makes the application of classic Diebold-Li method impossible or difficult over a longer time to maturity. This is particularly relevant because one of the main applications of yield curve estimation is forecasting. That is, embedding the estimation of yield curves in a dynamic framework that allows forecasting, is of utmost importance. Yet, this problem has rarely been tackled in the literature. One of the few exceptions in the work of Loechel et al. (2016) on the Chinese bond market. They estimate independent Nelson and Siegel (1987) period by period OLS models of the yield curve. In other words, they utilize the entire yield curve data (as we do), but contrary to Diebold-Li and to our approach, they ignore the dynamics when estimating the yield curves. Only in a second step, they
use the regression coefficients from their daily models (that correspond to the factors in a Diebold-Li framework) and to run a vector autoregressive model that allows predicting the yield curve.

We aim to fill this gap in the literature. Based on models originally developed for missing observations, we propose a simple augmentation of the Diebold et al. (2006) framework, which is more efficient than OLS estimation as it allows exploiting information from all available bonds and the time dependency of yields. The remainder of the paper is structured as follows. In Section 2 we first introduce the basic Diebold-Li framework, and then continue to derive our extension based on this foundation. Section 3 will discuss several empirical examples. We will start with a detailed discussion of the Chinese case in subsection 3.1 (to allow comparison to Loechel et al. (2016)). We continue to briefly present examples from four other major emerging markets in subsection 3.2 to show the general relevance of our approach. We use the other BRICS economies, Brazil, India and South Africa, as examples. Due to a lack of reliable data, we have to drop Russia. Instead, we add Indonesia, one of the largest emerging markets not included in the BRICS umbrella.

2 Method

2.1 The dynamic yield curve model

Diebold et al. (2006) use a simple AR(1) model of the Nelson-Siegel factors (level \( L \), slope \( S \) and curvature \( C \)), explaining a vector of yields \( r \) of different maturities \( \tau = [\tau_1 \tau_2 \ldots \tau_M] \)
\[
\begin{bmatrix}
  r_t(\tau_1) \\
  r_t(\tau_2) \\
  r_t(\tau_3) \\
  \vdots \\
  r_t(\tau_M)
\end{bmatrix}
= \begin{bmatrix}
  1 & \frac{1-e^{-\lambda \tau_1}}{\tau_1 \lambda} & \frac{1-e^{-\lambda \tau_1}}{\tau_1 \lambda} & -e^{-\tau_1 \lambda} \\
  1 & \frac{1-e^{-\lambda \tau_2}}{\tau_2 \lambda} & \frac{1-e^{-\lambda \tau_2}}{\tau_2 \lambda} & -e^{-\tau_2 \lambda} \\
  1 & \frac{1-e^{-\lambda \tau_3}}{\tau_3 \lambda} & \frac{1-e^{-\lambda \tau_3}}{\tau_3 \lambda} & -e^{-\tau_3 \lambda} \\
  \vdots \\
  1 & \frac{1-e^{-\lambda \tau_M}}{\tau_M \lambda} & \frac{1-e^{-\lambda \tau_M}}{\tau_M \lambda} & -e^{-\tau_M \lambda}
\end{bmatrix}
\begin{bmatrix}
  L_t \\
  S_t \\
  C_t
\end{bmatrix}
+ \varepsilon_t = H
\begin{bmatrix}
  L_t \\
  S_t \\
  C_t
\end{bmatrix}
+ \varepsilon_t \quad (1)
\]

\[
\begin{bmatrix}
  L_t \\
  S_t \\
  C_t
\end{bmatrix}
= \begin{bmatrix}
  \mu_L \\
  \mu_S \\
  \mu_C
\end{bmatrix}
+ \begin{bmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
  L_{t-1} \\
  S_{t-1} \\
  C_{t-1}
\end{bmatrix}
+ \eta_t = \mu + A
\begin{bmatrix}
  L_{t-1} \\
  S_{t-1} \\
  C_{t-1}
\end{bmatrix}
+ \eta_t, \quad (2)
\]

where \( \varepsilon_t \sim \mathcal{N}(0, R) \) and \( \eta_t \sim \mathcal{N}(0, Q) \) with \( R \) and \( Q \) being the covariance matrices of shocks to the observed yields and to the underlying states respectively, with \( R \) usually being assumed to be diagonal.

That is, this framework requires a fixed grid of maturities \( \tau = [\tau_1 \tau_2 \ldots \tau_M] \) over time, where the interest rate for each maturity included in the model is constantly available. However, in many emerging markets the bond markets are still developing. Often, only a limited number of bonds is traded. Therefore, to obtain yields at fixed maturities requires heavy interpolation, which unfortunately increases measurement uncertainty.\(^1\) Alternatively, one might apply a modified version of the state

\(^1\)Often this uncertainty is ignored and the interpolated values are treated as if they were actual measurements. Like any generated regressor problem that is unaccounted for, this can lead to a substantial underestimation of standard errors.
space framework that allows for missing observations as proposed by El-Shagi and Jiang (2017). Yet in this case, the missing observations cause massive identification problems in particular for the covariance matrix in the observation equation, and eventually – when too much data is missing – convergence can no longer be guaranteed (see e.g. Liu and Goldsmith, 2004). Therefore, other authors have reverted to the simple cross-sectional Nelson and Siegel (1987) approach, which allows exploiting the information from every bond available at a specific time by simply estimating the yield curve for a specific time via OLS (see e.g. Loechel et al., 2016). However, this comes at the cost of ignoring the time dimension.

2.2 Full information estimation of the dynamic model

Any yield \( r_t(\tau_m) \), \( m \in \{1, 2, \ldots, M\} \), in the Diebold et al. (2006) model is not the yield of a specific bond, but the yield associated with a specific maturity \( \tau_m \) at time \( t \). When estimating the equations 1 and 2 in their model, one typically assumes a diagonal covariance matrix \( R \) where the diagonal elements of \( R \) are unrestricted. That is, there is a different variance for every maturity \( \tau_m \). This is rather different from the assumption made when estimating yield curves via OLS in cross-sectional slices, which imposes constant variance across maturities. However, since the OLS estimation is independently repeated for each point in time, the variance is implicitly allowed to vary over time.

To understand the estimation problem when extending the Diebold et al. (2006)

\footnote{It is generally possible to estimate a state space model with missing observations. For example, this is frequently done in the mixed frequency literature and for high-frequency interpolation. See for example Mönch and Uhlig (2005).}
model to a version that incorporates all bond yields (rather than averaged bond yields for specific maturities) it helps to think about it in terms of a related problem. Starting from a standard version of the model with typically around 20 maturities, we extend the grid to distinguish more and more individual maturities. Eventually, this would lead to a situation where (in most situations) the yield for a specific maturity would be based on a single bond, and every single bond would be used to “compute” the yield for the matching maturity. 3 What happens when we gradually go into this direction is that the dimension of the yield vector $r$ and correspondingly the covariance matrix $R$ increase. That is, we have to estimate increasingly more coefficients. However, – unless there is an almost infinite variety of bonds – this will eventually also lead to more and more missing observations since many maturities considered will not be available for every observation. That is, the coefficients in $R$ have less and less support in the data, and their estimation is eventually infeasible. To overcome this problem, we have to impose restrictions on $R$ in order to keep the estimation possible.

In order to utilize the information of all available bond observation, we propose to combine the original (cross-sectional) Nelson and Siegel (1987) assumption with the panel setup (observing individual bonds over time) and treat the variance of $\varepsilon$ as constant both over maturities/bonds (as in Nelson-Siegel) and across time (as done for each maturity in the state space setup) simultaneously.4 This allows generalizing

3It is not identical, because several bonds with the same maturity might exist and still – hypothetically - have different yields in an imperfect market.

4Since a considerable proportion of the variation of uncertainty in the Diebold et al. (2006) model comes from the fact that we pool a different number of bonds to obtain the yield for a specific maturity and we are, contrarily, looking at individual bonds in our paper, this assumption is less restrictive than it looks at first glance.
the measurement equation to:

\[
    r_{i,t} = \left[ 1 - \frac{1 - e^{-\lambda \tau(i,t) \lambda}}{\tau(i,t) \lambda} - e^{-\tau(i,t) \lambda} \right] \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \varepsilon_{i,t},
\]

(3)

where \( \tau(i, t) \) is the time to maturity of bond \( i \) at time \( t \). Computationally, this requires to dynamically update \( H \) in equation 1 over time, because first, the maturity of each bond is changing over time (and thus the entries of \( H \) have to be updated) and second, our panel is unbalanced and thus the number of observations (and correspondingly the size of \( H \)) changes between points in time. Otherwise, our approach is a standard Kalman filter setup. The autoregressive coefficients of the state equation \( A \) (using the naming convention from Equation 2) and the (unconditional) covariances matrix of \( \eta \) and the (now scalar) variance of \( \varepsilon \) are estimated through maximum likelihood estimation using an expectation maximization algorithm for optimization.

While being easy to implement, this method allows massive efficiency gains for markets with sparse information compared with the Nelson and Siegel (1987) approach, as we demonstrate using the examples of emerging market data in next section.

Besides, if our assumption regarding the constant variance over time is correct, our model is much more efficient than individual estimation period by period via OLS. However, if there is heteroscedasticity over time, this assumption renders our approach inconsistent. This can easily be assessed with a standard Hausman test, where we simultaneously compare the set of estimated coefficients \( \tilde{\beta} = \)
\[
\begin{bmatrix}
\tilde{L}_1 \tilde{S}_1 \tilde{C}_1 \tilde{L}_2 \tilde{S}_2 \tilde{C}_2 \ldots \tilde{L}_T \tilde{S}_T \tilde{C}_T
\end{bmatrix}
\]
with the estimates from the state space estimation
\[
\hat{\beta} = \begin{bmatrix}
\hat{L}_1 \hat{S}_1 \hat{C}_1 \hat{L}_2 \hat{S}_2 \hat{C}_2 \ldots \hat{L}_T \hat{S}_T \hat{C}_T
\end{bmatrix}.
\] Since inverting a \(3T \times 3T\) matrix is infeasible for large \(T\), we ignore the covariance of estimation errors over time. In other words, we treat the covariance matrix of \(\tilde{\beta} - \hat{\beta}\) as block diagonal. The Hausman test can then be computed as
\[
\sum_{t=1}^{T} \left( \begin{bmatrix}
\tilde{L}_t \tilde{S}_t \tilde{C}_t
\end{bmatrix} - \begin{bmatrix}
\hat{L}_t \hat{S}_t \hat{C}_t
\end{bmatrix} \right) (\tilde{V}_t - \hat{P}_t) \left( \begin{bmatrix}
\tilde{L}_t \tilde{S}_t \tilde{C}_t
\end{bmatrix} - \begin{bmatrix}
\hat{L}_t \hat{S}_t \hat{C}_t
\end{bmatrix} \right)',
\]
where \(\hat{P}_t\) is the conditional covariance of the states in period \(t\) obtained from the Kalman smoother and \(\tilde{V}_t\) is the covariance estimate from the OLS estimator run for period \(t\).

### 3 Empirical Examples

#### 3.1 China

The Chinese bond market was massively extended in 2008 when more bonds of various maturity start to be regularly issued. Still, in the initial years after this major reform, there were large gaps in the available maturities at any point in time, resulting in exactly the problems discussed in the previous sections. Therefore, estimating the Chinese yield curve provides an excellent performance test for our method. The Chinese onshore bond market is split into two separate markets, the interbank market where major banks trade, and the exchange market where most of the mid-sized agents trade. For our empirical exercise, we focus on the exchange market. We use daily data from January 2008 to December 2016 with maturities up to 10 years.

Figure 1 shows the evolution of the estimated Chinese yield curve over time. We find the expected pattern with a yield curve, which is (usually) increasing in the
maturity, in a reasonable order of magnitude and with a low-interest rate period right after the financial crisis.

![Chinese yield curves from 2008 to 2016](image)

Figure 1: Chinese yield curves from 2008 to 2016

More important for our technical argument are the comparisons between our and OLS estimated yield curves on selected dates presented in Figure 2. While
producing similar point estimates, our estimates (in black) are much more efficient than the OLS estimates given in red. This is not only true in the earlier part of the sample, where the maturity of bond observations at each point in time is very sparse, but still for the more developed market in 2016 when more maturities are available. The Hausman test confirms the picture painted by the figures. In other words, we fail to reject the null hypothesis that the estimators of our state space approach are consistent. Besides, if we run individual Hausman tests for each day in our sample, that is to compare the set of estimated coefficients $\beta_t = [\tilde{L}_t, \tilde{S}_t, \tilde{C}_t]$ with the estimates from the state space estimation $\hat{\beta}_t = [\hat{L}_t, \hat{S}_t, \hat{C}_t]$ for any $t = 1, 2, ..., T$, there is no single rejection at the 5% level either.\(^5\)

### 3.2 Selected BRICS and emerging countries

In order to assess whether our method is suitable for a wider range of cases, we apply it to several other emerging countries. We consider three further BRICS economies, namely Brazil, India, and South Africa,\(^6\) and one of the major emerging economies with a large population, Indonesia. We use four years of trading records of domestically traded sovereign bonds of India, South Africa, and Indonesia, all denominated in local currency. Due to data availability, only a 150-day trading day window is covered for Brazil. For each of the four countries, we repeat the procedure described in subsection 3.1. That is, we estimate the yield curves over the respective sample periods with two methods (daily OLS estimates and our dynamic model), and then

\(^5\)We find one rejection out of more than 2300 tests at the 10% level.

\(^6\)All data for those economies are obtained from EIKON DataStream. The available data for Russia is insufficient for a meaningful quantitative exercise.
compare the results to assess the efficiency improvement. Figure 3 shows the dynamics of the yield curves of Indonesia, Brazil, India, and South Africa respectively, while Figure 4 provides snapshots comparing the two estimates (including confidence bounds) for all four countries on Feb 3rd, 2017. In all cases, our estimation generates narrower confidence intervals (in black) than the cross-sectional OLS estimation (in red), and this holds true over the whole sample period for all examples. Therefore, our examples confirm that our estimation is more efficient than the cross-sectional OLS estimation. For three out of four cases the Hausman test supports our model. We do, however, reject consistency for Brazil, indicating that daily OLS might be the more appropriate method in this one case. Brazil seems to have a very stable yield curve for most of our sample. This high persistency for most of the sample causes oversmoothing during the few periods when changes happen, driving the OLS estimate and the state space estimate apart.

One caveat of the dynamic approach is that periods of extremely high variance affect the estimate of unconditional variance in the Kalman filter. This increases uncertainty over the entire sample. If we extend the estimation for Brazil to the period where data gets even sparser (up to a point where daily OLS estimates for that period are not even possible), we start covering a period of extremely volatile financial markets. This will increase confidence bounds for all yield curves estimated through the state space model, bringing efficiency roughly to the level of independent OLS estimates.
4 Conclusion

In this paper, we propose an innovate approach based on Diebold et al. (2006) to estimate the emerging market yield curves, which is impossible or difficult because the maturity of outstanding bonds is very sparse in emerging markets. Compared with the alternative remedy, namely a simple cross-sectional OLS estimation, our modified Diebold-Li method produces more efficiency. We demonstrate those efficiency gains for four BRICS economies (Brazil, India, China, and South Africa) and Indonesia, i.e., some of the most relevant emerging economies. More importantly, our method could make a dynamic Nelson-Siegel model augmented with macroeconomic and financial variables available in investigating or forecasting emerging economies as in their developed counterparts. Our work would make the emerging market study more aligned with the frontier of yield curve related macroeconomics and forecasting research.

References


Figure 2: Selected Chinese yield curves from 2008 to 2016 every 190 trading days. 

*Note:* The black line is estimated with our version of the state space model. The red line is estimated with a cross sectional OLS approach. The solid line is the point estimator. Dotted lines represent the 90% confidence bounds.
Figure 3: Yield curves of selected emerging markets. Note: Due to data availability, our estimation covers only 150 trading days of Brazil whereas 4 years for the other three countries.
Figure 4: Yield curves of selected emerging markets on Feb 3rd, 2017. Note: The black line is estimated with our version of the state space model. The red line is estimated with a cross sectional OLS approach. The solid line is the point estimator. Dotted lines represent the 90% confidence bounds.